Applied Mathematics and Computation 252 (2015) 397–404

Contents lists available at ScienceDirect

Applied Mathematics and Computation

journal homepage: www.elsevier.com/locate/amc

Bifurcation and chaos in a discrete physiological control system

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ARTICLE INFO

Keywords:
Physiological system
Discrete model
Chaos

ABSTRACT

In this paper, we introduce a new method to discrete a physiological control system with delay and investigate the dynamical behavior of the discrete model. Basic properties of the discrete system are analyzed by means of Lyapunov exponent spectrum and bifurcation diagrams. Analysis results show that this system has complex dynamics with some interesting characteristics which may provide some new knowledge for us in fields of physiology.

1. Introduction

In recent years, there has been an more and more interest in the dynamical systems corresponding to physiological disorders for which a generally stable control system becomes unstable [1,6,7,10,12]. In this paper, we consider a hematopoiesis model with mature circulating cells [10]:

$$\frac{dP(t)}{dt} = \frac{\beta_0 \theta^m}{\theta^m + P^m(t-\tau)} - \gamma P(t), \tag{1}$$

where P represents the density of mature cells in blood circulation, τ represents the time delay between the production of immature cells in the bone marrow and their maturation for release in circulating bloodstreams, $\beta_0,\ \theta$ and γ are all positive constants.

There has been some work on the stability analysis of the model (1). Berezansky and Braverman presented the persistence and extinction conditions of model (1) with variable coefficients and a nonconstant delay [2]. Rost considered the global attractivity of the positive equilibrium for the delay Eq. (1) [15]. Krisztin and Liz investigated the bubbles behavior of model (1) [8]. Kanno and Uchida introduced a method for the calculation of finite-time Lyapunov exponents in model (1) and obtained standard deviation of the probability distribution of the finite-time Lyapunov exponents [5].

However, the advantages of a discrete-time approach are multiple in physiological system [25,14,9,19]. Firstly, the statistics are compiled from given time intervals and not continuously and thus difference models are more realistic than differential ones. Secondly, the discrete-time models can provide natural simulators for the continuous cases (i.e., differential models). As a result, discrete hematopoiesis models are studied by many scholars.

Wang and Li studied globally dynamical behaviors for discrete approximation of model (1) with almost periodic coefficients [21]. Su and Ding investigated the dynamics of Mackey–Glass system applied a nonstandard finite-difference scheme and obtained hopf bifurcations [16]. Neimark–Sacker and fold bifurcations was shown in the difference equation of model (1) [17]. Qian considered global attractivity of periodic solutions of model (1) in discrete from [13].

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http://dx.doi.org/10.1016/j.amc.2014.11.107
0096-3003/

In this paper, we introduce a discrete method for model (1). Based on Lyapunov exponent spectrum and bifurcation diagrams, we found that such difference model has rich and complex dynamical behaviors including bifurcation and chaos. Some discussion and conclusion are given in the last section.

2. A discrete model

In the following part, we use semi-discrete method to obtain a discrete approximation of model (1). Let $P(t) = \theta x(t)$, then we get that:

$$\frac{dx(t)}{dt} = -\gamma x(t) + \frac{\beta}{1 + x^m(t-\tau)}, \tag{2}$$

where $\beta = \beta_0/\theta$.

Let $[\cdot]$ denote the greatest integer function, the following delay differential equation with piecewise constant arguments

$$\frac{dx(t)}{dt} = -\gamma x(t) + \frac{\beta}{1 + x^m[t-\tau]} \tag{3}$$

can be viewed as a semi-discretization of system (2).

As a result, we have

$$\frac{d[x(t)e^{\gamma t}]}{dt} = \frac{\beta e^{\gamma t}}{1 + x^m[t-\tau]}. \tag{4}$$

We integrate (4) from the positive integer n to $n+1$ and obtain

$$x_{n+1} = \frac{1}{e^{\gamma}} x_n + \frac{\beta}{\gamma}\left[\frac{\sigma e^{-\gamma}(e^{\gamma\sigma} - 1)}{1 + x^m_{n-k-1}} + \frac{(1-\sigma)\left(1 - e^{\gamma(\sigma-1)}\right)}{1 + x^m_{n-k}}\right], \tag{5}$$

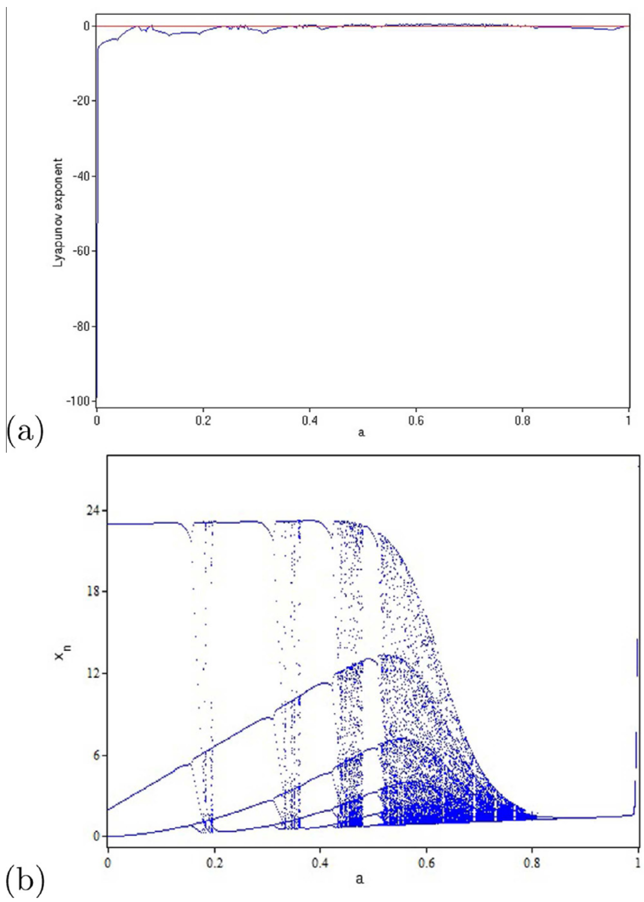

Fig. 1. Spectrum of Lyapunov exponents and bifurcation diagrams for $a - x_n$ with $b = 10,\ c = 23$ and $m = 15$.