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Numerical bifurcation and its application in computation of available transfer capability ☆

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ABSTRACT

Numerical bifurcation analysis techniques are very powerful and efficient in physics, biology, engineering, and economics. This paper mainly discusses its application in computation of available transfer capability (ATC) among the specific interface in AC–DC hybrid power grid. Considering the load as a parameter, which implies the load increment at any bus and the specified generation schedule, the saddle node bifurcation and the Hopf bifurcation have been used for static and dynamic ATC determination respectively. The proposed method has been applied to a modified WSCC 9-bus AC–DC power system for the ATC determination. The impact of the method has also been discussed.

1. Introduction

Bifurcation analysis is the study of behavioral variations within families of nonlinear systems, which are usually defined as parameter-dependent sets of ordinary differential equations (ODEs) or differential and algebraic equations (DAEs). The numerical bifurcation analysis is concerned with the stable, reliable and efficient computation of solutions for nonlinear parameterized problems. Here we consider the parameter-dependent ODEs of the form

$$\dot{x} = f(x, \lambda), \tag{1.1}$$

where f is a nonlinear smooth operator in an appropriate Banach space setting, $x \in \mathbb{R}^n$ is a vector of state variables and $\lambda \in \mathbb{R}$ represents a parameter. Examples of systems of the form (1.1) are ubiquitous in mathematical modeling. The nonlinearity of the equations gives rise to possibly complicated dynamics as the parameter varies.

Because of the difficulties in using analytical methods, numerical computation has been widely used as a main tool to detect bifurcation points (x^*, λ^*) of nonlinear systems (1.1). Since H.B. Keller developed the numerical analysis of continuation methods in the late 1970s [1], numerical bifurcation analysis has become a well established mathematical tool for the nonlinear analysis of multifarious models arising from a wide range of problems, although we shall concentrate on problems arising in determination of ATC in AC–DC power systems. There are a comprehensive account of numerical methods and codes for solving bifurcation problems that have been developed (one can refer to [2] and references therein for a detail).

For generic one-parameter problems, eigenvalues of the Jacobian on the imaginary axis appear in two ways: one is a simple zero eigenvalue, and the other is a conjugate pair $\pm i\omega, \omega > 0$, of purely imaginary eigenvalues. Corresponding to the first

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singularity, two solutions coalesce and annihilate each other under parameter variation, it appears generically as a limit point, this kind of bifurcation is also referred to in the technical literature as a fold or turning points. The second singularity is corresponding to a Hopf (or Andronov-Hopf) bifurcation (HB) in generical, from which a family of periodic solutions emerges. These two types of bifurcation are generically the bifurcations we expect to see for one parameter system. For this reason they are called co-dimension one bifurcations. Numerical computation of bifurcations in large equilibrium systems in MATLAB is introduced in [3]. It is well known that singular points of parameter-dependent problems are often regular points of certain extended systems. Numerous regular augmented systems have been constructed to compute viable singular points, such as folds, transcritical bifurcation points, pitchfork bifurcation points, Hopf bifurcation points, etc [2].

Power system engineering has become a classical application field of bifurcation theory since 1990's. Various types of problems on power systems can be modeled as either a set of parameterized ODEs or a set of parameterized DAEs. Local bifurcations are readily evident in power systems as important elements of voltage instability [4–6]. There are three common types of local bifurcation phenomena emerging from the equilibrium point, namely saddle node bifurcation (SNB), HB and singularity induced bifurcation (SIB). From a point of electrical engineer's view, the SNB happens with a bifurcation of equilibrium points, is one important form of voltage instability. This phenomenon is also called fold or limit point bifurcation. During slow changes of parameters (usually the real and reactive power injections) SNB can lead to voltage collapse. The HB occurs when a pair of complex conjugate eigenvalues moves from the left to right half of the complex plane crossing the imaginary axis at points other than the origin. HB has been associated with an emergence of oscillatory instability or dynamic voltage instability. While the SIB is a phenomenon attributed exclusively to DAEs. It is the case that the equilibria undergo stability exchanges and one of the eigenvalues of the Jacobian matrix becomes unbounded. The occurrence of the SIB can be obtained by inspecting the existence of singular sets in the state space containing singular points for each parameter value.

The available transfer capability (ATC) is a significant signal that refers to the capability of a system to transport or deliver energy above that of already subscribed transmission uses. In order to realize open access to electric power transmission networks and promote generation competition and customers' choice, ATC should be calculated hourly, daily or monthly based on market requirements. Along with the nationwide power network extensively interconnecting by high voltage direct current (HVDC) power systems, the influence of HVDC should be considered in computation of ATC for AC–DC hybrid power systems. However, few works has been done on this issue [8,9]. And to the best of the authors knowledge, since bifurcation theory was first introduced to determinate ATC in the paper [10], no work has been reported so far which utilises the bifurcation approach for determination of dynamic ATC in AC–DC hybrid power systems. Apart from considering the static limits, application of bifurcation analysis would be a novel idea in the computation of dynamic ATC. We'll consider the techniques of numerical bifurcation for determination of ATC, hereafter static ATC has been determined using the SNB limit, and the dynamic ATC has been computed using the HB limit as in the paper [10]. The proposed approach has been demonstrated on a modified WSCC 9-bus system.

The rest of this paper is organized as follows. In Section 2, we give a brief review of some definitions and the augmented systems for corresponding singular points. The modeling of ATC is described in Section 3 and numerical bifurcation method to determine the static and dynamic ATC is proposed. In Section 4, we demonstrate the proposed approach on a modified WSCC 9-bus system. At the last section, we summarize the results.

2. A Survey of numerical bifurcation theory

In this section we will formulate conditions defining the fold and the Hopf bifurcations of equilibria in n-dimensional continuous-time systems. Then the augmented systems for solving these bifurcations are reviewed.

Considering the problem (1.1), we use the notation $f_x(x,\lambda), f_\lambda(x,\lambda), f_{xx}(x,\lambda), f_{x\lambda}(x,\lambda), f_{\lambda\lambda}(x,\lambda)$ to denote partial Frechét-derivatives of f at $(x,\lambda) \in \mathbb{R}^n \times \mathbb{R}$.

2.1. Fold bifurcation

Definition 1 [2]. A point (x^*,λ^*) is a fold point of $f(x,\lambda)$ with respect to λ if

$$f(x^*,\lambda^*) = 0, \tag{2.1}$$

$$\ker f_x(x^*,\lambda^*) \neq \{0\}, \tag{2.2}$$

$$f_\lambda(x^*,\lambda^*) \notin \mathit{range}\, f_x(x^*,\lambda^*). \tag{2.3}$$

In addition, a fold point (x^*,λ^*) is a simple fold of $f(x,\lambda)$ if

$$\dim \ker f_x(x^*,\lambda^*) = \mathit{codim\ range}\, f_x(x^*,\lambda^*) = 1. \tag{2.4}$$

A direct method to compute the fold point is solving the following set of 2n + 1 equations augmented by additional equations that characterize the bifurcation points

$$F(x,\phi,\lambda) = \begin{pmatrix} f(x,\lambda) \\ f_x(x,\lambda)\phi \\ l^T\phi - 1 \end{pmatrix} = 0, \tag{2.5}$$