Applied Mathematics and Computation 250 (2015) 12–27

Contents lists available at ScienceDirect

Applied Mathematics and Computation

journal homepage: www.elsevier.com/locate/amc

Optimal matrix pencil approximation problem in structural dynamic model updating ☆

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ARTICLE INFO

Keywords:
Optimal matrix pencil approximation
Alternating direction method
Structure-preserving
Model updating

ABSTRACT

The problem of finding the least change adjustment to a given matrix pencil is considered in this paper. Desired matrix properties including satisfaction of characteristic equation, symmetry, positive semidefiniteness, and sparsity are imposed as side constraints to form the optimal matrix pencil approximation problem. Such a problem is related to the frequently encountered engineering problem of a structural modification on the dynamic behavior of a structure. Alternating direction method is applied to this constrained minimization problem. Numerical results are included to illustrate the performance and application of the proposed method.

1. Introduction

It is well known that the analytical model of a real-life structure, obtained by the finite element technique, may be represented by the following dynamic equation (characteristic equation) or generalized eigenvalue problem

$$K_a x = \lambda M_a x, \tag{1}$$

where $M_a,\ K_a \in R^{n\times n}$ denote the analytical mass and stiffness matrices, respectively, n is the number of degrees of freedom of the finite element model, and λ and x are the eigenvalue and corresponding eigenvector with respect to the matrix pencil (M_a, K_a). In general, M_a and K_a are symmetric positive definite and semidefinite sparse matrices with special zero/nonzero pattern, respectively. Due to the complexity of engineering structures, the finite element models fail to reproduce the dynamic behavior of actual structures accurately. Hence, the finite element model should be updated and maintain the topological structure of the original model.

Over the past years, finite element model updating problem has received much attention. Various methods have been developed for correcting analytical models to predict test results more closely [1–3]. One of the most common methods in structural dynamic model updating is the optimal matrix update methods, in which the measured data are assumed to be exact and the stiffness or/and mass matrices to be corrected should satisfy the system's relationships with minimal deviation from the finite element model [4–29]. Most of the existed methods just take part of the positive semidefiniteness and the sparsity requirement of the updated matrix into consideration. Recently, Yuan [30,31] considered the problem of updating the analytical stiffness matrix to satisfy with the dynamic equation, symmetry, positive semidefiniteness and sparsity simultaneously, proposed the matrix linear variational inequality approach and proximal-point method for solving the problem by using partial Lagrangian multipliers technique.

☆ The work is supported by the Fundamental Research Funds for the Central Universities No. 56XAA14084.
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http://dx.doi.org/10.1016/j.amc.2014.10.099
0096-3003/

In this paper, both the mass and stiffness matrices are updated to satisfy with the dynamic equation, symmetry, positive semidefiniteness and sparsity simultaneously. Such a problem is formulated as the following matrix pencil nearness problem

$$\begin{aligned}
&\min \frac{c}{2}\|M-M_a\|_F^2+\frac{1}{2}\|K-K_a\|_F^2\\
&s.t.\ KX_e = MX_e\Lambda_e,\\
&M^T = M \succeq 0,\\
&K^T = K \succeq 0,\\
&\text{sparse}(M)=\text{sparse}(M_a),\\
&\text{sparse}(K)=\text{sparse}(K_a),
\end{aligned} \tag{2}$$

where Λ_e is a nonsingular diagonal matrix and $X_e \in R^{n\times m}$ is of full column rank, $m < n$. In general, the magnitude of the stiffness term $\|K-K_a\|_F$ is far greater than that of the mass term $\|M-M_a\|_F, c$ is a scaling factor to narrow the gap. $\text{sparse}(M)=\text{sparse}(M_a)$ means that the mass matrix to be corrected should have the same zero/nonzero pattern as the matrix M_a, and so does $\text{sparse}(K)=\text{sparse}(K_a)$.

Note that problem (2) is a minimization of a proper strictly convex quadratic function over the intersection of a finite collection of closed convex sets in $R^{n\times n}$, namely, a convex programming problem. Liao et al. [40] discussed the best approximate solution of matrix equation $AXB+CYD=E$ without the sparsity constraint, which is similar to problem (2). Due to the positive semidefiniteness and sparsity constraints, it is difficult to express the elements in the feasible region of problem (2) explicitly. Furthermore, the constraints on M and K can be equivalently transformed into separable constraints on (M,K). Hence, Problem (2) can be solved equivalently via a separable convex programming problem, to which alternating direction method (ADM) can be applied, which was proposed by Gabay and Mercier [32] to solve separable convex programming. In recent years, alternating direction method has received extensive attention for its applications [33–36,39,41].

Now we introduce some notations for further discussion. I is the identity matrix of appropriate order in context. For $A,B\in R^{m\times n}$, an inner product in $R^{m\times n}$ is defined by $\langle A,B\rangle=\text{trace}(B^TA)$, then $R^{m\times n}$ is a Hilbert space. The matrix norm $\|\cdot\|_F$ induced by the inner product is the Frobenius norm. S_+^n is the set of all $n\times n$ symmetric positive semidefinite matrices. e_i is the ith column of the identity matrix I and P is an appropriate permutation matrix defined by $P=[e_{i_1},e_{i_2},\ldots,e_{i_n}]$, where $(i_1,i_2,\ldots,i_n)$ is a permutation of $(1,2,\ldots,n)$.

The remainder of this paper is arranged as follows. In Section 2, the original alternating direction method is briefly reviewed and a basic assumption is made. In Section 3, we focus on dealing with two subproblems of the equivalent form of Problem (2). In Section 4, two engineering examples in structure dynamic model updating are performed by our proposed method. Conclusions are given is Section 5.

2. Preliminaries

For completeness, we first review ADM [32] briefly.

Let $G_1: R^s\to(-\infty,+\infty]$ and $G_2: R^t\to(-\infty,+\infty]$ be closed proper convex functions. A is a $t\times s$ matrix, and C_1 and C_2 are nonempty closed convex subsets of R^s and R^t respectively. Consider the following problem

$$\begin{aligned}
&\min G_1(y)+G_2(z)\\
&s.t.\ Ay-z=0,\\
&y\in C_1, z\in C_2.
\end{aligned} \tag{3}$$

Let $\lambda\in R^t$ be the Lagrange multiplier vector and r be a positive parameter which penalizes for the violation of the constraint. The augmented Lagrangian of Problem (3) is

$$L_r(y,z,\lambda)=G_1(y)+G_2(z)+\langle\lambda,Ay-z\rangle+\frac{r}{2}\|Ay-z\|_2^2. \tag{4}$$

Given $y^{(k)}, z^{(k)}$ and $\lambda^{(k)}$, the iteration scheme of ADM may be described as

$$\begin{cases}
y^{(k+1)}=\underset{y\in C_1}{\text{argmin}}\ \{G_1(y)+\langle\lambda^{(k)},Ay\rangle+\frac{r}{2}\|Ay-z^{(k)}\|_2^2\},\\
z^{(k+1)}=\underset{z\in C_2}{\text{argmin}}\ \{G_2(z)-\langle\lambda^{(k)},z\rangle+\frac{r}{2}\|Ay^{(k+1)}-z\|_2^2\},\\
\lambda^{(k+1)}=\lambda^{(k)}+r[Ay^{(k+1)}-z^{(k+1)}].
\end{cases} \tag{5}$$

The efficiency of ADM depends on the solutions of $y^{(k+1)}$ and $z^{(k+1)}$ in Eq. (5). If $rank(A)=s, y^{(k+1)}$ and $z^{(k+1)}$ are always uniquely attained. Hence, ADM is well defined for Problem (3). By the definition of the Frobenius norm, the vector type ADM can be extended directly to the matrix type one. In addition, due to the positive semidefiniteness and the sparsity constraints, it is difficult to give the conditions for guaranteeing the non-emptiness of the feasible region of Problem (2), denoted by D. All thought this paper, we assume that D is nonempty.