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Model reduction of a class of discrete-time nonlinear systems

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ABSTRACT

This paper considers the problem of model reduction of a class of discrete-time systems subject to Lipschitzian nonlinearities. It is shown that under some conditions the nonlinear system can be either approximated by a discrete-time linear time-invariant system or a nonlinear system of reduced order. The computation of the matrices of the reduced-order system is carried out through the solutions of a set of linear matrix inequalities. The proposed design is approved by the simulation of reduced-order dynamics of a mass-spring system subject to a nonlinear friction and a linear electric circuit with uncertain parameters.

1. Introduction

Model reduction has received a widespread attention since many decades due to its importance in circuit simulation [1,2], feedback design, image processing and other engineering fields, see e.g., [3,4]. The reduced order system might be used as a component in a larger simulation or to develop a low dimensional controller suitable for real time applications. The idea behind model reduction is to replace the original system by a reduced-order one such that the input–output behaviors are maintained close to each other. Quite successful methods have been implemented, including but not limited to: balanced-truncation algorithms, see e.g. [5], Moment matching techniques [3], projection-based procedures, optimal and convex-optimization techniques, see e.g., [6–10].

In order to capture and preserve the basic properties of the large-order original system while its reduction to a minimal order system, it is required to define certain criterions guaranteeing bounded error approximation. The H_∞ norm of the difference of two transfer functions is one of the most meaningful measures of the approximation error, see e.g., [8]. H_2-norm minimization-based algorithms [11], convex-optimization-based techniques [10,9,12,], and Hankel model-reduction-based procedures [13] have shown their good performances for a variety of dynamical systems. The reader is also referred to the references [14–16,3,17,5,18] for different looks on model reduction using Kalman's minimal realization and balanced truncation procedures. Recent optimization techniques as numerical genetic algorithms and Particle Swarm Optimization-based procedures have been successfully applied to model reduction and identification, see e.g., [19] and the references therein. Further details on available techniques for model reduction are largely discussed in the survey paper [4] and the references therein. Even though model reduction procedures are developed rather properly for linear dynamical systems, there are still many issues in their generalization to the nonlinear case, see e.g., [20–26]. Among the difficulties that appear in nonlinear model reduction are the absence of general methods that assure global approximation with predefined absolute error, the complexity of the generalization of balancing methods to nonlinear systems, and the problem of stability

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preserving by projection. Even many different approaches to nonlinear model reduction have been developed for continuous time nonlinear systems, see e.g., [21,23,25,26], the topic of extension of the obtained results to discrete-time nonlinear systems remains an attractive area of research.

In this paper, we deal with model reduction of a class of stable discrete-time nonlinear systems subject to Lipschitzian nonlinearities. Examples of such systems include but not limited to: stable Hamiltonian mechanical systems subject to bounded frictions, stable chemical reactions, and distillation-column systems with rational bounded nonlinearities [27]. As a first objective, it is shown that it is quite possible to approximate a discrete-time high-order nonlinear system by a low-order *linear time-invariant system* if the system under consideration is asymptotically stable for the null control input and globally bounded for any bounded control input. This type of approximation is quite useful for simplification of controller design and system approximation when nonlinearities are poorly known. The design of the reduced-order system is conditioned by the solvability of a set of linear matrix inequalities that must hold simultaneously. The second major result of the present paper is to lay down a systematic nonlinear model reduction technique, guaranteeing both stability of reduced order systems and absolute-error approximation. The second model-reduction technique is based on combining Krylov-projection model-reduction method with convex-optimization procedures giving rise to a nonlinear-model-reduction procedure guaranteeing a global absolute error bound. This method does not require nonlinearity approximation or expansion but, in contrast to the first developed procedure, it requires the well knowledge of the system dynamics. An example of a fourth-order mechanical system with nonlinear friction and an example of an electrical circuit are studied to approve the efficiency and the usefulness of the proposed numerical techniques.

2. System approximation by reduced-order linear systems

2.1. Preliminary results

Throughout this paper, we note by $\mathbb{R}, \mathbb{N}$, and $\mathbb{Z}_{\geqslant 0}$ the set of real numbers, the set of natural numbers, and the set of positive integer numbers, respectively. The notation $A > 0$ (resp. $A < 0$) means that the matrix A is positive definite (resp. negative definite). A' is the matrix transpose of A. We note by $\|\cdot\|$ the usual Euclidean norm. $\|G(z)\|_\infty$ refers to the infinity norm of a discrete-time transfer function $G(z)$ as $\|G(z)\|_\infty = \sup_{\omega\in[0,2\pi]}\sigma_{\max}(G(e^{j\omega}))$, where $\sigma_{\max}$ represents the maximum singular value of matrices. The space of square summable functions over the interval $[0, N-1]$ is denoted by $\mathscr{L}_2[0, N-1]$ and $\|u_k\|_2 = (\sum_0^{N-1} u_k' u_k)^{\frac{1}{2}}$. $\mathbf{0}$ and I stand for the null matrix and the identity matrix of appropriate dimensions while I_n and $0_{n,n}$ stand for the identity matrix of dimensions $n \times n$ and the null matrix of dimensions $n \times n$, respectively. We note by $\mathrm{Col}(A)$ the set of columns of the matrix A. The symbol "$\star$" in any matrix stands for any element that is induced by transposition. The following Lemmas are needed for the proof of the main statements.

Lemma 2.1 [28.] *For any constant symmetric matrix $M \in \mathbb{R}^{n\times n}, M = M' > 0$, scalar $\gamma > 0$, vector function $\omega : [0, \gamma] \mapsto \mathbb{R}^n$ such that the integration in the following is well defined, we have*

$$\gamma \int_0^\gamma \omega'(\beta) M \omega(\beta) \mathrm{d}\beta \geqslant \left(\int_0^\gamma \omega(\beta)\mathrm{d}\beta\right)' M \left(\int_0^\gamma \omega(\beta)\mathrm{d}\beta\right). \tag{1}$$

Lemma 2.2 (*The Schur complement lemma [29]*). *Given constant matrices M, N, Q of appropriate dimensions where M and Q are symmetric, then $Q > 0$ and $M + N'Q^{-1}N < 0$ if and only if*

$$\begin{bmatrix} M & N' \\ N & -Q \end{bmatrix} < 0, \text{ or equivalently } \begin{bmatrix} -Q & N \\ N' & M \end{bmatrix} < 0.$$

Lemma 2.3 [30]. *The following two statements are equivalent:*

(1) *A is stable and $\|C(zI - A)^{-1}B + D\|_\infty < \gamma$.*
(2) *There exists a symmetric matrix $P > 0$ satisfying*
$A'PA - P + C'C + (A'PB + C'D)R^{-1}(B'PA + D'C) < 0$ *where* $R = \gamma^2 I - B'PB - D'D$.

In the next subsection, necessary conditions for model-reduction of nonlinear systems by reduced-order linear systems are given.

2.2. Necessary conditions for nonlinear model reduction with absolute-error constraint

Consider the nonlinear discrete-time system:

$$\begin{aligned} x_{k+1} &= A x_k + f(x_k) + B u_k, \\ y_k &= C x_k, \end{aligned} \tag{2}$$