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Takeover time in Evolutionary Dynamic Optimization: From theory to practice

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ABSTRACT

Making theoretical has been a hard task for researchers in the field of Evolutionary Dynamic Optimization (EDO), as only a few approaches have appeared in recent years. In EDO, problems change over time, requiring from the solver, an Evolutionary Algorithm (EA), to continuously adapt to new conditions. Mathematical tools such as the *takeover time* models, extensively used to characterize and compare EAs in static problems, become much more difficult to understand when the problem changes over time. A preliminary takeover time model have been recently introduced for tournament selection and diversity-generating approaches. In this article, we propose a new enhanced model that takes into account important scenarios that were not initially considered. We use predictive modeling to describe the EAs performance and statistical analysis to validate our equations. Finally, we show how these theoretical models can be used to build novel techniques in EDO.

1. Introduction

Evolutionary Dynamic Optimization (EDO) has become an important topic in the last two decades as many real-world optimization problems change over time [1–3]. Typical examples include dealing with new arrivals (in Job shop scheduling), minimizing response times (to changing customer demand), or connecting moving nodes (on mobile networks), among others. These problems are known in the literature as Non-Stationary, Time-varying, or Dynamic Optimization Problems (DOP), and are formally defined by an objective function which is deterministic over certain time periods, called *stationary intervals*, that is dependent on time, i.e.:

$$F(X) = f_t(X) \tag{1}$$

where f_t is a stationary objective function within the t-th time interval, and parameter X represents a candidate solution. The goal here is to find the optimal solution O_t for each function $f_t(X)$.

DOPs challenge standard Evolutionary Algorithms (EAs) since they tend to converge to a single point of the search space, and the loss of diversity prevents them from reacting and adapting to new environmental conditions. Several approaches have been developed to address this limitation: generating diversity after a change [4], maintaining the population diversity throughout the run [5], using memory to re-use data from past [6,7], considering multiple populations to search and track many optimal candidates at the same time [8], or involving learning and prediction [9], also based on the experience gained in previous scenarios. Moreover, there are many hybrid schemes that combine one or more of these techniques [10,7,11].

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However, the success of EAs largely depends on the *selection pressure* [12], i.e., the extent to which better individuals are favored by a selection method, since this determines the convergence speed, and a very fast/slow convergence can cause the algorithm to fail. This feature has been widely studied in static problems using several metrics: *takeover time* [13], *selection intensity* [14], *loss of diversity* [14,15], and *reproduction rate* [16]. However, much must still to be done regarding these theoretical studies in EDO.

Recently, the concept of takeover time (the time that it takes for the best individual to completely fill the population under selection only) was extended to EDO by Bravo et al. [17], as the expected value at each stationary interval (see Section 2). Authors also introduced a preliminary takeover time model for tournament selection and reactive approaches (those that generate diversity in response to a change in the environment), based on the *change severity* (how different each new function is with respect to the previous one) and the *change period* (how often the problem changes), two main features of DOPs.

However, the initial takeover time model for EDO does not consider the number of local optima, another key feature when modeling the EAs' performance in DOPs. This is important since it affects the number of times the current best solution continue being optimum after a change, one of the main estimations of the initial model. In addition, to simplify the analysis, such a model was limited to change periods longer than the expected takeover time. There are many possible scenarios where that condition is not met, derived from the complementary case where the problem changes before the best solution completely fills the population. In these scenarios the takeover time model depends on both the number and size of the fitness classes in the population, which are hard to estimate.

Solving all these limitations is one of our main contributions in this article. With this aim we build completely new predictive equations to also take the new scenarios into account. We use a comprehensive test-bed benchmark (much larger than the one used in the preliminary work) and perform statistical analysis to validate the new approaches. Furthermore, we not only build theoretical models of takeover time, but we also use them to enhance the reactive approach for EDO by showing how new solvers can be created. The performance improvement of this new techniques crafted with our takeover time model will show a new reason to work in this theory to practice line of research of algorithms backed by theory.

The remainder of this article is organized as follows. Section 2 analyzes the basic concepts and examines a base takeover time model in EDO, points out its limitations and proposes a new takeover time model to overcome them. Section 3 validates the accuracy of the new model in comparison to the previous models. Section 4 uses the new model to propose a new reactive approach for EDO. Finally, conclusions and future work are given in Section 5.

2. Takeover time in EDO

Theoretical models of *growth curves* and *takeover time* have been widely used to model the EA behavior. In static problems, the *growth curves* induced by a given selection method is defined as a function $P_t : \mathbb{N} \rightarrow (0, 1]$ that maps the proportion of copies of the best solution in the population to the generation step. This definition assumes the existence of a single copy of the best solution in the initial population ($P_0 = 1/n$, for a population size n) and selection only (without variation operators). As a result, in each generation the number of copies of the current best solution grows. The number of generation steps it takes for the EA to completely fill the population of copies of the best solution is called *takeover time*, and denoted by t^*:

$$t^* = \min\{t : P_t = 1\} \tag{2}$$

where P_t is the function defining the growth curve for the selection method used.

In DOPs, the *growth curve* has been seen to increase within each stationary interval and crumbles after a change, showing different behaviors along the run (see Fig. 1). The reader can distinguish two different scenarios, *A* and *B*, after an environmental change occurs (every 10 generations). However, the figure suggests that after an environmental change the scenario *B* occurs more times than the scenario *A* throughout the run, for the case of the DOP instance tested.

Therefore, the takeover time for EDO is defined as the expected value of takeover time at each stationary interval, and denoted by $\hat{t}$:

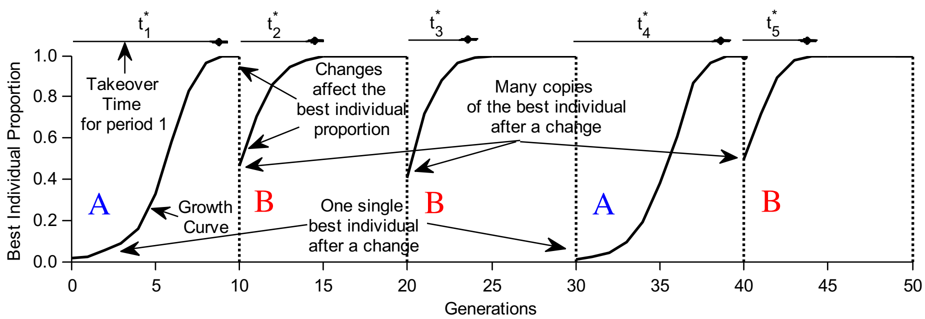

Fig. 1. Growth curve in DOPs ($\tau \geqslant t^*$). We distinguish two scenario types: *A* and *B*.