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Exponential stability criterion for interval neural networks with discrete and distributed delays

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ABSTRACT

This paper investigates the global exponential stability of neural networks with discrete and distributed delays. A new criterion for the exponential stability of neural networks with mixed delays is derived by using the Lyapunov stability theory, Homomorphic mapping theory and matrix theory. The obtained result is easier to be verified than those previously reported stability results. Finally, some illustrative numerical examples are given to show the effectiveness of the proposed result.

1. Introduction

Neural networks (NNs) have been widely studied in the past few decades, due to their practical importance and successful applications in many areas, such as image processing, combinatorial optimization, signal processing, pattern recognition, associative memories, and so on. It has been recognized that time delay often exists in neural network model because neurons cannot communicate and respond instantaneously. In 1989, time delay is introduced into Hopfield neural networks by Marcus and Westervelt [1]. Afterward, many literatures studied the stability of systems, especially for neural network, such as [2–36]. Up to now, various methods have been employed to get stability of neural networks (NNs) with discrete delay in the previous literatures, for example, linear matrix inequality method [3], norm inequality technique [6], and, etc. However, as pointed out in [2], it should be noted that neural networks (NNs) usually have spatial extent due to the presence of a multitude of parallel pathways with a variety of axon sizes and lengths. In recent years, neural networks (NNs) with discrete and distributed delays has attracted much attention in research [10–15,18]. Many methods have been proposed to reduce the conservatism of the stability criteria, such as model transformation method, free-weighting matrix approach, constructing novel Lyapunov functionals method, delay decomposition technique, and weighting-matrix decomposition method. For example, by using Lyapunov functional, modified stability condition for neural networks with discrete and distributed delays has been obtained in [18]; by constructing general Lyapunov functional and convex combination approach, the stability criterion for neural networks with mixed delays has been obtained in [11]; by partitioning the time delay and using Jensen integral inequalities, the stability condition on delayed neural networks with both discrete and distributed delays has been obtained in [10]. It should be noted that the discrete delay $\tau(t)$ was constrained on $\dot{\tau}(t) < 1$ in [10,11]. Obviously, the constraint was quite strong, and it can be relaxed to some extent. Furthermore, to the best of our knowledge, there are not any result on the neural networks with discrete and distributed delays by using the M-matrix

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method and Homomorphic mapping theory. Therefore, there are rooms for further improvement in stability analysis of neural networks with discrete and distributed delays.

In this paper, a novel stability condition for neural networks with discrete and distributed delays is obtained by the Lyapunov stability theory, Homomorphic mapping theory and M-matrix theory. The obtained result improves conditions in [10–14,18]. The sufficient criterion in this paper does not need the differentiability of the discrete delay $\tau(t)$. Finally, some illustrative numerical examples are given to illustrate the effectiveness and the advantage of the proposed criteria.

Notation: In this paper, $A \leqslant B$ means that corresponding elements of the matrices A and B satisfy $a_{ij} \leqslant b_{ij}$, where $A = (a_{ij})_{n\times n}$ and $B = (b_{ij})_{n\times n}$; I is identity matrix; $\|\cdot\|$ denotes a vector or a matrix norm; Superscript 'T' denotes transposition of a vector or a matrix; R and R^n are real and n-dimension real number sets, respectively; For a vector $x = (x_1, x_2, \ldots, x_n)^T$ and a matrix $A = (a_{ij})_{n\times n}$, $|x| = (|x_1|, |x_2|, \ldots, |x_n|)^T$, $|A| = (|a_{ij}|)_{n\times n}$; λ_{min} denotes the minimum eigenvalue of a real matrix.

2. Preliminaries

Consider the interval neural networks with discrete and distributed delays with the following ordinary differential model:

$$\dot{x}_i(t) = -c_i x_i(t) + \sum_{j=1}^{n} a_{ij} f_j(x_j(t)) + \sum_{j=1}^{n} b_{ij} f_j(x_j(t-\tau_j(t))) + \sum_{j=1}^{n} d_{ij} \int_{t-\sigma}^{t} f_j(x_j(s))ds + u_i, \quad i = 1,2,\ldots,n. \tag{1}$$

Rewritten it in the vector form as:

$$\dot{x}(t) = -Cx(t) + Af(x(t)) + Bf(x(t-\tau(t))) + D\int_{t-\sigma}^{t} f(x(s))ds + U, \tag{2}$$

where $x(t) = (x_1(t), x_2(t), \ldots, x_n(t))^T$ denotes the neuron state vector; $C = diag(c_1, c_2, \ldots, c_n)$ is a positive diagonal matrix; $A = (a_{ij})_{n\times n}$ is the interconnection weight matrix and $B = (b_{ij})_{n\times n}$, $D = (d_{ij})_{n\times n}$ are the delayed interconnection weight matrices, respectively; $f(x(t)) = (f_1(x_1(t)), f_2(x_2(t)), \ldots, f_n(x_n(t)))^T$ denotes the neuron activations; $f(x(t-\tau(t))) = (f_1(x_1(t-\tau_1(t))), f_2(x_2(t-\tau_2(t))), \ldots, f_n(x_n(t-\tau_n(t))))^T \in R^n$; $U = (u_1, u_2, \ldots, u_n)^T$ denotes the constant input vector; $\tau_i(t) (i = 1,2,\ldots,n)$ denotes the discrete delay, which is assumed to be bounded and nonnegative functions, i.e., there exists a positive number τ, such that $0 \leqslant \tau_i(t) \leqslant \tau$; σ is distributed delay.

In order to completely characterize the equilibrium and stability properties of the neural networks model (2), the parameter uncertainties are inevitable. Taking uncertainties parameters a_{ij}, b_{ij}, c_i in the model, it is assumed that they have bounded norms in the following intervals:

$$\begin{aligned}
C_I &:= \{C = diag(c_i) : 0 < \underline{C} \leqslant C \leqslant \overline{C}, \text{ i.e., } 0 < \underline{c}_i \leqslant c_i \leqslant \overline{c}_i, \quad \forall i = 1,2,\ldots,n\},\\
A_I &:= \{A = (a_{ij}) : \underline{A} \leqslant A \leqslant \overline{A}, \text{ i.e., } \underline{a}_{ij} \leqslant a_{ij} \leqslant \overline{a}_{ij}, \quad i,j = 1,2,\ldots,n\},\\
B_I &:= \{B = (b_{ij}) : \underline{B} \leqslant B \leqslant \overline{B}, \text{ i.e., } \underline{b}_{ij} \leqslant b_{ij} \leqslant \overline{b}_{ij}, \quad i,j = 1,2,\ldots,n\},\\
D_I &:= \{D = (d_{ij}) : \underline{D} \leqslant D \leqslant \overline{D}, \text{ i.e., } \underline{d}_{ij} \leqslant d_{ij} \leqslant \overline{d}_{ij}, \quad i,j = 1,2,\ldots,n\},
\end{aligned} \tag{3}$$

where $\underline{C} = diag(\underline{c}_i), \overline{C} = diag(\overline{c}_i)$, $\underline{A} = (\underline{a}_{ij})_{n\times n}, \overline{A} = (\overline{a}_{ij})_{n\times n}$, $\underline{B} = (\underline{b}_{ij})_{n\times n}, \overline{B} = (\overline{b}_{ij})_{n\times n}$, $\underline{D} = (\underline{d}_{ij})_{n\times n}, \overline{D} = (\overline{d}_{ij})_{n\times n}$.

Denote

$$\begin{aligned}
A^* &= \frac{1}{2}(\overline{A} + \underline{A}), \quad A_* = \frac{1}{2}(\overline{A} - \underline{A}),\\
B^* &= \frac{1}{2}(\overline{B} + \underline{B}), \quad B_* = \frac{1}{2}(\overline{B} - \underline{B}),\\
D^* &= \frac{1}{2}(\overline{D} + \underline{D}), \quad D_* = \frac{1}{2}(\overline{D} - \underline{D}).
\end{aligned} \tag{4}$$

It is obviously that A_*, B_*, D_* are nonnegative matrices, and the interval matrices $[\underline{A}, \overline{A}], [\underline{B}, \overline{B}], [\underline{D}, \overline{D}]$ are equivalent to $[A^* - A_*, A^* + A_*], [B^* - B_*, B^* + B_*], [D^* - D_*, D^* + D_*]$, respectively. So, the following formulations hold

$$\begin{aligned}
A &= A^* + \Delta A, \quad \Delta A \in [-A_*, A_*],\\
B &= B^* + \Delta B, \quad \Delta B \in [-B_*, B_*],\\
D &= D^* + \Delta D, \quad \Delta D \in [-D_*, D_*].
\end{aligned} \tag{5}$$

The neuron activation functions $f_i(x_i)$ satisfies the following condition

$$|f_i(x_i) - f_i(y_i)| \leqslant l_i|x_i - y_i|, \quad i = 1,2,\ldots,n, \ \forall x_i, \ y_i \in R, \tag{6}$$

where $l_i (i = 1,2,\ldots,n)$ is known constant scalar.