Applied Mathematics and Computation 250 (2015) 193–211

Contents lists available at ScienceDirect

Applied Mathematics and Computation

journal homepage: www.elsevier.com/locate/amc

Global dynamics of an additional food provided predator–prey system with constant harvest in predators

Moitri Sen [a], P.D.N. Srinivasu [b], Malay Banerjee [a,*]

[a] *Department of Mathematics and Statistics, I.I.T. Kanpur, Kanpur 208016, India*
[b] *Department of Mathematics, Andhra University, Visakhapatnam 530003, India*

ARTICLE INFO

Keywords:
Stable coexistence
Oscillatory coexistence
Extinction
Local bifurcation
Global bifurcation

ABSTRACT

The article aims to study the global dynamics associated with a predator prey system when the predator is provided with additional food and harvested at a constant rate. This study supplements the existing literature on the dynamics of additional food provided predator prey system by focusing on the consequences of harvesting the predators. It presents a comprehensive view on the entire range of bifurcations that take place in the considered system and highlights the dependence of the system dynamics on its vital parameters. This study provides important tools for investigations pertaining to controllability of the system which are essential from the real world applications perspective.

1. Introduction

Dynamics of predator–prey systems when the predator is provided with additional food has become a topic of intense study due to its wide applications in biological control and species conservation. Some theoretical studies [2,20,28] conclude that provision of additional food to predators enhances the predator density which can influence the predator in a way that increases target predation resulting in lowering the density of target prey. This reduction in prey density due to presence of alternative food for predators is known to be *apparent competition* [19]. Whereas empirical studies indicate that provision of additional food to predators need not always increase target predation [16,20,24,37]. This apparent conflict between theory and observations led to in depth mathematical analysis of additional food provided predator–prey systems.

Under the assumption that the additional food is uniformly distributed in the habitat and the number of encounters per predator with the additional food is proportional to the density of the additional food, it is observed that additional food serves as a biological control agent with which both prey as well as predators can be controlled. The nutritive value and the quantity of the additional food play vital role in the controllability of the predator–prey system [31,32,35]. By and large, it is observed that provision of additional food of high nutritive value could increase target predation and reduce the prey density and, additional food with low nutritive value could release the prey from predation pressure and decrease predator density.

The recent works [27,31–34], present a comprehensive account on the dynamics associated with predator–prey systems in presence of additional food to predators. They also bring out controllability of the said predator–prey system using the quality (nutritive value) and quantity of the additional food as control parameters. These works not only supplement the understanding on pest management using biological means but also find their applicability in biological conservation and

---

* Corresponding author.
E-mail address: malayb@iitk.ac.in (M. Banerjee).

http://dx.doi.org/10.1016/j.amc.2014.10.085

resource management problems. Therefore, availability of the studies on effect of additional food on sustainability of the involved species leaves us with an important problem of assessing the considered system dynamics under harvesting which is the focus in the present article.

Dynamics of predator–prey models with constant-yield harvesting (harvested biomass is independent of the population size) and constant-effort harvesting (a constant proportion of the population is harvested) are studied extensively by several authors [3,4,6–8,13,21,38]. Predator–prey models with the harvesting of either prey or predators or both the species have been studied as the outcomes of such analysis are helpful for the management of renewable resources [1,5]. Effect of harvesting on the dynamic behavior of the concerned model is not only interesting from theoretical point of view but also important for the management and sustainability of commercially/economically important resources [10–12,14,15,18,21]. Systematic analysis of harvested predator–prey models indicate the chance of extinction of one or both the species due to uncontrolled harvesting. Further, it determines the admissible level of harvesting to ensure the long term survival of the renewable resource.

In this paper, we bring forward a systematic study on the dynamics of additional food provided predator–prey system with constant harvesting (also known as ‘constant-yield harvesting’) in predators. Dynamics of predator–prey system with constant harvesting in predators is presented in [39]. Essentially, this work presents complete bifurcation analysis for the considered model. Similar kind of dynamical analysis has been carried out for predator–prey systems in [26,29,36]. This analysis unfolds the rich dynamics exhibited by the predator–prey system when the predator is provided with additional food and also harvested at a constant rate. It is observed that constant harvesting brings in more complexity into the system dynamics when compared with the systems which are (i) free from harvesting; (ii) subjected to constant-effort harvesting [21,22]. This is due to inclusion of more number of equilibrium solutions and involvement of global bifurcation such as homoclinic bifurcation and codimension two local bifurcation namely Bogdanov–Takens bifurcation. Existence of such kind of bifurcations always indicate the over exploitation of resource and leading to the extinction scenario.

The section-wise division of this paper is as follows. In the next section we describe the model representing dynamics of a predator–prey system when the predator species is provided with additional food and simultaneously harvested with constant harvesting. Section 3 presents analysis pertaining to the existence of equilibria and their local stability nature. Local bifurcation analysis and global dynamics of the system are presented in Sections 4 and 5 respectively. Section 6 presents an over view of the variety of dynamics of the considered system. Finally, Section 5 presents discussion and conclusions.

2. Basic model

Let N and P represent biomass of prey and predator respectively. Let us assume that the predator is provided with additional food of biomass A which is distributed uniformly in the habitat. If $h_1(h_2), e_1(e_2), n_1(n_2)$ respectively represent the handling time of the predator per unit quantity of prey (additional food), ability of the predator to detect the prey (additional food) and the nutritive value of the prey (additional food) then the following system describes the predator–prey dynamics in presence of additional food to predators.

$$\frac{dN}{dT} = rN\left(1-\frac{N}{K}\right) - \frac{e_1 NP}{1+e_1h_1N+e_2h_2A}, \tag{2.1a}$$

$$\frac{dP}{dT} = \frac{n_1e_1NP + n_2e_2AP}{1+e_1h_1N+e_2h_2A} - mP, \tag{2.1b}$$

where r and K respectively stand for intrinsic growth rate and carrying capacity of the prey population, and m is the death or starvation rate of the predator. Defining $c = \frac{1}{h_1}, b = n_1c, a = \frac{1}{e_1h_1}, \eta = \frac{n_2e_2}{n_1e_1}$ and $\alpha = \frac{n_1h_2}{n_2h_1}$, the system (2.1) takes the form

$$\frac{dN}{dT} = rN\left(1-\frac{N}{K}\right) - \frac{cNP}{a+\alpha\eta A+N}, \tag{2.2a}$$

$$\frac{dP}{dT} = \frac{b(N+\eta A)P}{a+\alpha\eta A+N} - mP \tag{2.2b}$$

with all the involved parameters being positive. Here the parameters a, b and c stand for half saturation value of the predator in the absence of additional food, maximum birth rate of the predator due to consumption of the food perceived by the predator and maximum rate of capturing of prey by predator, respectively. Observe that the above system assumes Holling type II predator functional response towards its available food and that the number of encounters per predator with the additional food is proportional to additional food biomass. Further, this model reduces to the well known Rosenzweig–MacArthur model [5] in the absence of additional food to predators.

The model (2.2) has been analyzed from biological control perspective [31] and studies pertaining to controllability of the system with respect to the parameters α and η have been presented in [32,34]. In the present study we wish to investigate consequences of harvesting the predators at a constant rate where in the predators are provided with additional food. Incorporating constant yield harvesting (ρ) into the model (2.2) we obtain the following system: