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Global dynamics of a predator, weaker prey and stronger prey system

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ABSTRACT

In this paper, we propose and analyze a prey–predator system consisting of two competitive prey populations and one predator population which depends on both the prey species. We investigate the boundedness and persistence criteria of the system and existence conditions of all the possible equilibria. Further the dynamical behavior from the point of view of local and global stability at different equilibria are presented. We also determine the explicit conditions so that the system has no periodic solutions. Finally, we present some numerical examples to illustrate our analytical works.

1. Introduction

The relationship between prey and their predator populations is one of the dominant field in theoretical ecology as well as in applied mathematics. To describe the ultimate dynamical behavior between prey and predator, models can be a very useful and powerful tool. Different prey–predator interactions like, pest and their natural enemies, plankton-fish interactions, etc. can be described through proper mathematical models. Many authors used mathematical models with the help of ordinary differential equations to describe the prey–predator system. Goh [1], Hastings [2], Korobeinikov [3], Aziz-Alaoui and Okiye [4], Kar [5], Zhang et al. [6], Kar and Ghorai [7] etc. effectively used different mathematical models to describe the interactions between prey and the predator species. Some more theoretical works on different types of dynamical behaviors of prey–predator system can be observed in Yan and Liu [8], Zhang et al. [9], Jana and Kar [10,11], Kar and Jana [12], Chakraborty et al. [13] and references therein.

In all prey–predator systems, there is a common competition between prey and predator species and in general this competition is one sided i.e. the loss occurs only in the prey populations. Further in most of the works on the dynamics of prey–predator system, the authors in general consider single predator depending on single prey. In ecosystem analysis through mathematical models, some authors have also considered two predator-single prey system (see, [14–18]) and few researchers have considered an ecosystem where one predator population depends on two or more prey populations (see [19–22]). It is also a common phenomenon to observe in some biological systems that a predator species depends on two different prey species. For example, in a pest control problem the natural enemy can be considered as the predator population of two different types of pest (prey) populations, namely, the harmful pest or target pest species and non target pest species [23]. In our present work we consider the interaction of one predator species depending on two different types

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of prey species. Particularly, in this article we divide the whole live food (i.e. prey) of a predator species into two prey classes namely the stronger prey class and the weaker prey class. To construct the mathematical model of this system, we assume that the predator species do not need any handling time or searching time for the weaker prey species. On the other hand for the stronger prey species it is considered that predator population needs some handling time as well as searching period. To include these issues in the model system, we use the mass action law for the interaction between weak prey and predator and Holling type II functional response for the interaction between stronger prey and predator. To make our model more realistic, we consider competitions among all the individuals present in the system. This competition may be either between two individuals of different species (inter-specific) or between two individuals of same species (intra-specific).

Now to form the mathematical model, we take the biomass density of stronger prey as x_1, the weaker prey as x_2 and the predator as y. We consider logistic type growth for both the prey species with intrinsic growth rate and environmental carrying capacity respectively as r_1 and k_1 for the x_1 species and r_2 and k_2 for the x_2 species. Now since x_2 is assumed to be comparatively weaker prey population and having no handling time period for the predator species, therefore we assume that predator species (y) predates x_2 species according to Holling type-I functional response with β be the maximum predation rate. Similarly we assume predator captures x_1 species according to Holling type-II functional response with maximum predation rate α and half saturation constant a. Further we assume that the respective proportions m and η from the x_1 and x_2 species contribute to the growth of the predator. Therefore the three dimensional differential equation for two prey one predator system is as follows:

$$\begin{aligned}
\frac{dx_1}{dt} &= r_1x_1\left(1-\frac{x_1}{k_1}\right)-\frac{\alpha x_1 y}{a+x_1},\\
\frac{dx_2}{dt} &= r_2x_2\left(1-\frac{x_2}{k_2}\right)-\beta x_2 y,\\
\frac{dy}{dt} &= \frac{m\alpha x_1 y}{a+x_1}+\eta\beta x_2 y-\delta y.
\end{aligned} \tag{1.1}$$

As we pointed out earlier, competition between two different species or within a single species has an important role in ecosystem (see [24,25]), in our present work we modify our model (1.1) by introducing the competition between the weaker and the stronger prey class and assume that n_1 and n_2 are their respective death rates due to competitions. Furthermore we consider the intra-specific competition within the predator species y and due to this competition, their per capita death rate is taken as μ. Hence the proposed two prey one predator model is as follows:

$$\begin{aligned}
\frac{dx_1}{dt} &= r_1x_1\left(1-\frac{x_1}{k_1}\right)-\frac{\alpha x_1 y}{a+x_1}-n_1x_1x_2,\\
\frac{dx_2}{dt} &= r_2x_2\left(1-\frac{x_2}{k_2}\right)-\beta x_2 y-n_2x_1x_2,\\
\frac{dy}{dt} &= \frac{m\alpha x_1 y}{a+x_1}+\eta\beta x_2 y-\delta y-\mu y^2,
\end{aligned} \tag{1.2}$$

subject to the initial conditions

$$x_1(0)\geqslant 0,\quad x_2(0)\geqslant 0,\quad y(0)\geqslant 0. \tag{1.3}$$

Here all the parameters are positive and having their own usual meaning. The rest part of the paper is organized as follows. In Section 2, we provide the boundedness of all the solutions of the system (1.2). In Section 3 we find all the feasible equilibria of the system and their local stability analyses are given in Section 4. Section 5 is devoted to examine the non existence of periodic solution of the system and in Section 6, we describe global behavior at different equilibria. Some numerical simulations are given in Section 7, to verify the theoretical results and in penultimate section we discuss the main findings of the article. Finally, in last section we summarize our paper.

2. Boundedness of solutions

In this section we now describe the uniform boundedness of the solutions of the system (1.2).

Theorem 2.1. *The solutions of the system (1.2) are always uniformly bounded.*

Proof. Let us construct a function

$$B = x_1+\frac{\eta}{m}x_2+\frac{1}{m}y. \tag{2.1}$$