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Numerical solutions of system of linear Fredholm–Volterra integro-differential equations by the Bessel collocation method and error estimation

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ABSTRACT

In this study, the Bessel collocation method is presented for the solutions of system of linear Fredholm–Volterra integro-differential equations which includes the derivatives of unknown functions in integral parts. The Bessel collocation method transforms the problem into a system of linear algebraic equations by means of the Bessel functions of first kind, the collocation points and the matrix relations. Also, an error estimation is given for the considered problem and the method. Illustrative examples are presented to show efficiency of method and the comparisons are made with the results of other methods. All of numerical calculations have been made on a computer using a program written in Matlab.

1. Introduction

Integro-differential equation systems (IDES) are encountered in model problems of science and engineering. Since it is usually difficult to find analytical solutions of IDES, a numerical scheme is required. In recent years, many authors for the systems of the integral and integro-differential equations have worked on numerical methods such as the Haar functions method [1,2], the homotopy perturbation method [3,4], the Legendre matrix method [5], the Lagrange method [6,7], the differential transformation method [8], the Tau method [9], the variational iteration method [10], the modified homotopy perturbation method [11], the Taylor collocation method [12], the Adomian method [13], the Galerkin method [14], the Chebyshev polynomial method [15], the Bernstein operational matrix approach [16] and the operational Tau method [17].

Lately, Yüzbaşı et al. [18–21] have studied the Bessel collocation method for system of differential equations, system of Volterra integral equations, a class of systems of Fredholm integro-differential equations, the pollution model of a system of lakes and the system of multi pantograph equations. Also, Yüzbaşı [22,23] has applied by developing the Bessel collocation method for the HIV infection model of CD4 + T cells and continuous population models for single and interacting species.

In this paper, we will apply the Bessel collocation method studied in [9,13–16] for the approximate solutions of the system of high-order linear Fredholm–Volterra integro-differential equations (FVIDEs) in the form

$$\sum_{n=0}^{m}\sum_{j=1}^{k}P_{i,j}^{n}(x)y_j^{(n)}(x)=g_i(x)+\int_a^b\sum_{n=0}^{m}\sum_{j=1}^{k}K_{i,j}^{n,f}(x,t)y_j^{(n)}(t)dt+\int_a^x\sum_{n=0}^{m}\sum_{j=1}^{k}K_{i,j}^{n,v}(x,t)y_j^{(n)}(t)dt,\quad i=1,2,\ldots,k, \tag{1}$$
$$0\leqslant a\leqslant x,\quad t\leqslant b,$$

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with the mixed conditions

$$\sum_{j=0}^{m-1}\left(a^n_{i,j}y_n^{(j)}(a)+b^n_{i,j}y_n^{(j)}(b)\right)=\lambda_{n,i},\quad i=0,1,\ldots,m-1,\quad n=1,2,\ldots,k, \tag{2}$$

where $y_j^{(0)}(x)=y_j(x)$ is an unknown function, $P^n_{i,j}(x),\ g_i(x),\ K^{n,f}_{i,j}(x,t)$ and $K^{n,v}_{i,j}(x,t)$ are the functions defined in the interval $a\leqslant x,t\leqslant b$, the functions $K^{n,f}_{i,j}(x,t)$ and $K^{n,v}_{i,j}(x,t)$ for $i,j=1,2,\ldots,k$ can be expanded Maclaurin series and also $a^n_{i,j},\ b^n_{i,j}$ and $\lambda_{n,i}$ are appropriate constant.

Our purpose is to obtain approximate solutions of (1) in the truncated Bessel series form

$$y_i(x)=\sum_{n=0}^{N}a_{i,n}J_n(x),\quad i=1,2,\ldots,k,\quad 0\leqslant a\leqslant x\leqslant b, \tag{3}$$

so that $a_{i,n},\ n=0,1,2,\ldots,N$ are the unknown Bessel coefficients, N is chosen any positive integer such that $N\geqslant m$, and $J_n(x),\ n=0,1,2,\ldots,N$ are the Bessel functions of the first kind defined by

$$J_n(x)=\sum_{k=0}^{[\![\frac{N-n}{2}]\!]}\frac{(-1)^k}{k!(k+n)!}\left(\frac{x}{2}\right)^{2k+n},\quad n\in\mathbb{N},\quad 0\leqslant x<\infty.$$

To found a solution in the (3) of the problem (1) with the conditions (2), we can use the collocation points defined by

$$x_s=a+\frac{b-a}{N}s,\quad s=0,1,\ldots,N,\quad 0\leqslant a\leqslant x\leqslant b. \tag{4}$$

2. Bessel collocation method for system of FVIDEs

Let us consider the system (1) and find the matrix forms of each term of the equation. We first consider the approximate solution $y_j(x)$ given in the system (3). We can put the approximate solution$y_j(x)$ in the matrix form

$$[y_j(x)]=\mathbf{J}(x)\mathbf{A}_j,\quad j=1,2,\ldots,k, \tag{5}$$

where

$$\mathbf{J}(x)=[\,J_0(x)\quad J_1(x)\quad\cdots\quad J_N(x)\,]\quad\text{and}\quad \mathbf{A}_j=[\,a_{j,0}\quad a_{j,1}\quad\cdots\quad a_{j,N}\,]^T.$$

ith-order derivative of this solution is

$$[y_j^{(i)}(x)]=\mathbf{J}^{(i)}(x)\mathbf{A}_j. \tag{6}$$

Now, we can write clearly the matrix form $\mathbf{J}(x)$ as follows

$$\mathbf{J}(x)=\mathbf{X}(x)\mathbf{D}^T, \tag{7}$$

where

$$\mathbf{X}(x)=\left[\,1\quad x\quad x^2\quad\cdots\quad x^N\,\right]\quad\text{and the coefficients matrix } \mathbf{D}[21].$$

i th-order derivative of the expression (7) is

$$\mathbf{J}^{(i)}(x)=\mathbf{X}^{(i)}(x)\mathbf{D}^T \tag{8}$$

and the relation between the matrix $\mathbf{X}(x)$ and its derivative $\mathbf{X}^{(i)}(x)$ is

$$\mathbf{X}^{(i)}(x)=\mathbf{X}(x)\left(\mathbf{B}^T\right)^i \tag{9}$$

such that

$$\mathbf{B}^T=\begin{bmatrix}0&1&0&\cdots&0\\0&0&2&\cdots&0\\\vdots&\vdots&\vdots&\ddots&\vdots\\0&0&0&\cdots&N\\0&0&0&\cdots&0\end{bmatrix}\quad\text{and}\quad \left(\mathbf{B}^T\right)^0=[\mathbf{I}]_{(N+1)\times(N+1)}\ \text{is unit matrix.}$$

By substituting Eq. (9) into the relation (8), we get the matrix representation

$$\mathbf{J}^{(i)}(x)=\mathbf{X}(x)\mathbf{B}^i\mathbf{D}^T. \tag{10}$$