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A numerical analysis of a center circular-hole crack in a rectangular tensile sheet

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ABSTRACT

By using a hybrid displacement discontinuity method (a boundary element method), a center circular-hole crack in a rectangular tensile sheet is investigated in this paper. Detail numerical solutions of the stress intensity factors (SIFs) of the circular-hole crack are given, which can reveal the effect of geometric parameters of the cracked body on the SIFs. By comparing the calculated SIFs of the circular-hole crack with those of the center crack, the shielding and amplifying effects of the circular hole on the SIFs are found. The effects have perhaps an important meaning in engineering.

1. Introduction

Due to the stress concentration, cracks are likely to initiate at these hole's boundaries under loading action. Consequently, a number of papers dealing with hole edge crack problems are available, see Ref. [1]. For radial crack(s) emanating from a circular hole in an infinite plate under tension, typical solutions were given by Bowie [2] and Newman [3]. For radial cracks emanating from an elliptical hole in an infinite plate under tension, typical solutions were obtained by Nisitani and Isida [4], by Murakami [5] by using a body force and by Newman [3] by using a boundary collocation method. For cracks emanating from a triangular or square hole in an infinite plate under tension, Murakami [5] used the body force method to calculate their stress intensity factors.

In this paper, a numerical analysis of a circular-hole crack in a rectangular plate in tension (see Fig. 1) is performed by using a hybrid displacement discontinuity method [6]. In Ref. [1], detailed solutions of the SIFs of the circular-hole crack shown in Fig. 1 are listed as H/W is so large that the effect of H/W on the SIFs is negligible. But to the author's knowledge, detail solutions to the SIFs of the circular-hole crack have not been obtained when H/W is smaller than 2. Specifically, detail numerical solutions of the SIFs of the circular-hole crack are given and discussed in this paper. By comparing the calculated SIFs of the circular-hole crack with those of the center crack, the shielding and amplifying effects of the circular hole on the SIFs are found. The effects have perhaps an important meaning in engineering.

2. Brief description of the hybrid displacement discontinuity method

In this section, the hybrid displacement discontinuity method presented by Yan [6] is described briefly. It consists of the constant displacement discontinuity element presented by Crouch and Starfield [7] and the crack-tip displacement discontinuity elements.

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2.1. Constant displacement discontinuity element (constant element)

The displacement discontinuity D_i is defined as the difference in displacement between the two sides of the segment [7] (see Fig. 2):

$$\begin{aligned} D_x &= u_x(x,0_-) - u_x(x,0_+) \\ D_y &= u_y(x,0_-) - u_y(x,0_+) \end{aligned} \tag{1}$$

The solution to the subject problem is given by Crouch and Starfield [7]. The displacements and stresses can be written as

$$\begin{aligned} u_x &= D_x[2(1-v)F_3(x,y) - yF_5(x,y)] + D_y[-(1-2v)F_2(x,y) - yF_4(x,y)], \\ u_y &= D_x[(1-2v)F_2(x,y) - yF_4(x,y)] + D_y[2(1-v)F_3(x,y) - yF_5(x,y)] \end{aligned} \tag{2}$$

and

$$\begin{aligned} \sigma_{xx} &= 2GD_x[2F_4(x,y) + yF_6(x,y)] + 2GD_y[-F_5(x,y) + yF_7(x,y)], \\ \sigma_{yy} &= 2GD_x[-yF_6(x,y)] + 2GD_y[-F_5(x,y) - yF_7(x,y)], \\ \sigma_{xy} &= 2GD_x[-F_5(x,y) + yF_7(x,y)] + 2GD_y[-yF_6(x,y)]. \end{aligned} \tag{3}$$

G and v in these equations are shear modulus and Poisson's ratio, respectively. Functions F_2 through F_7 are described in Ref. [7]. Eqs. (2) and (3) are used by Crouch and Starfield [7] to set up a constant displacement discontinuity method.

2.2. Crack-tip displacement discontinuity elements (crack-tip elements)

By using the Eqs. (2) and (3), recently, Yan [6] presented crack-tip displacement discontinuity elements, which can be classified as the left and the right crack-tip displacement discontinuity elements to deal with crack problems in general plane elasticity. The following gives basic formulas of the left crack-tip displacement discontinuity element.

For the left crack-tip displacement discontinuity element (see Fig. 3), its displacement discontinuity functions are chosen as

$$D_x = H_s\left(\frac{a_{tip}+\xi}{a_{tip}}\right)^{\frac{1}{2}}, \quad D_y = H_n\left(\frac{a_{tip}+\xi}{a_{tip}}\right)^{\frac{1}{2}}. \tag{4}$$

where H_s and H_n are the tangential and normal displacement discontinuity quantities at the center of the element, respectively, a_{tip} is a half length of crack-tip element. Here, it is noted that the element has the same unknowns as the two-dimensional constant displacement discontinuity element. But it can be seen that the displacement discontinuity functions defined in (4) can model the displacement fields around the crack tip. The stress field determined by the displacement discontinuity functions (4) possesses $r^{-1/2}$ singularity around the crack tip.

Based on the Eqs. (2) and (3), the displacements and stresses at a point (x,y) due to the left crack-tip displacement discontinuity element can be obtained,

$$\begin{aligned} u_x &= H_s[2(1-v)B_3(x,y) - yB_5(x,y)] + H_n[-(1-2v)B_2(x,y) - yB_4(x,y)], \\ u_y &= H_s[(1-2v)B_2(x,y) - yB_4(x,y)] + H_n[2(1-v)B_3(x,y) - yB_5(x,y)] \end{aligned} \tag{5}$$

and

$$\begin{aligned} \sigma_{xx} &= 2GH_s[2B_4(x,y) + yB_6(x,y)] + 2GH_n[-B_5(x,y) + yB_7(x,y)], \\ \sigma_{yy} &= 2GH_s[-yB_6(x,y)] + 2GH_n[-B_5(x,y) - yB_7(x,y)], \\ \sigma_{xy} &= 2GH_s[-B_5(x,y) + yB_7(x,y)] + 2GH_n[-yB_6(x,y)], \end{aligned} \tag{6}$$

where functions B_2 through B_7 are described in Ref. [6].

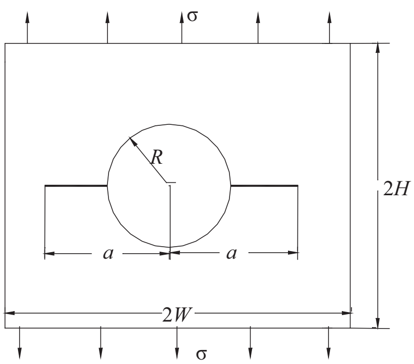

Fig. 1. Schematic of a circular-hole crack in a rectangular plate in tension.