Applied Mathematics and Computation 250 (2015) 390–401

Contents lists available at ScienceDirect

Applied Mathematics and Computation

journal homepage: www.elsevier.com/locate/amc

Stochastic stabilization of singular systems with Markovian switchings ☆

Guoliang Wang

School of Information and Control Engineering, Liaoning Shihua University, Fushun 113001, China

ARTICLE INFO

Keywords:
Singular Markovian jump systems
Stochastic stabilization
Partially unknown transition probabilities
Linear matrix inequalities (LMIs)

ABSTRACT

This paper studies the stochastic stabilization problem for a class of singular Markovian jump system. The aim is to determine whether or not there is a stochastic feedback controller stabilizing a given singular Markovian jump system (SMJS). A new kind of stochastic controller only in the diffusion part is proposed such that the closed-loop system has a unique solution and is almost surely exponentially admissible. New sufficient condition for the existence of such a controller is given as linear matrix inequalities (LMIs). Based on this, more extensions to transition probability matrix (TPM) with elements partially unknown and system states partially observable are developed. A numerical example is used to demonstrate the effectiveness of the proposed methods.

1. Introduction

Due to system parameters or structures changed abruptly, many practical systems [1–4] are usually modeled into Markovian jump systems (MJSs). Such systems have time-evolving and event-driven mechanisms simultaneously. The first one related to sate vector is continuous in time. The second one named as operation mode or system mode is driven by a Markov process taking values in a finite set. In the past decades, many important results have been obtained, see, e.g., [5–17]. On the other hand, singular systems [18,19] are convenient in description and control of electrical systems, economics, chemical processes and mechanics, etc. Compared with normal systems, two additional modes named as impulsive modes and non-dynamic modes respectively are included in singular systems. Owing to the essential differences between normal and singular systems, the latter systems are usually more complicated, in which the stability, regularity and impulse elimination (for continuous case) or causality (for discrete case) should be considered simultaneously. Up to now, a lot of attention has been paid to this system. Especially, singular Markovian jump systems (SMJSs) [20–22] have natural representation in the description of MJSs experiencing abrupt changes. In recent years, many research topics on SMJSs have been studied in [23–31].

As we know, environmental noise exists in many dynamical systems and cannot be neglected. Such systems are usually described as stochastic systems. When there are jump parameters, they are usually modeled into stochastic hybrid systems (SHSs). This kind of system has important applications in many areas, such as air traffic management [32], communication networks [33] and biology systems [34]. Because the stochastic models exist in many branches of science and industry, SHSs

☆ This work was supported by the National Natural Science Foundation of China under Grants 61104066, 61203001, 61374043 and 61473140, the China Postdoctoral Science Foundation funded project under Grant 2012M521086, the Program for Liaoning Excellent Talents in University under Grant LJQ2013040, the Natural Science Foundation of Liaoning Province under Grant 2014020106.

E-mail address: glwang985@163.com

http://dx.doi.org/10.1016/j.amc.2014.11.019
0096-3003/

have received a lot of attention and are widely used, see e.g., [35–39]. It is obtained from these references that the stabilization problem is realized by a regular controller in terms of being in the drift part. In references [40–43], it has been proved that a Brownian noise perturbation referred to a controller in the diffusion section can stabilize or destabilize a system which cannot be stabilized by a regular controller. If the underlying system has jumping parameters, the stochastic stabilization problem was considered in [44,45]. By investigating these references, it is seen that they are all about normal stochastic systems. It means that the derivative matrix is nonsingular and is always simplified to be an identical matrix. When the derivative matrix of SHSs is singular, they may be named as stochastic singular systems (SSSs), and very few results are concerned on such systems. The main reason is that the existence of such a singular derivative matrix makes the analysis and synthesis of SSSs quite different to normal SHSs. Even for the essential problems of singular systems such as the regularity and impulse elimination, there are very few results to report these issues. Recently, the authors in [46] considered the stability of SSSs with Markovian switchings, where a non-convex assumption on system matrices is needed to guarantee the uniqueness of solution. By applying a proportional-derivative state feedback controller (PDSFC), the authors in [47] discussed the robust control problem of stochastic SMJSs (SSMJSs), where the system matrices including the derivative matrix have uncertainties. The key idea of PDSFC is to transform the original SSMJS into a normal SHS. Then, the regularity and impulse elimination of singular systems are avoided, and the uniqueness of solution to SSMJSs is transformed to a case similar to normal SHSs. The desired PDSFC in particular is a regular controller and is in the drift part. Moreover, from the existing stochastic stabilization results on SHSs, it is seen that the transition probability plays important roles in stochastic stabilization and should be known exactly. This assumption is very ideal and will have the application scope largely limited. When the TPM is partially unknown that some elements of a TPM are inaccessible, the results of stochastic stabilization in the afore-said references will be failed. In this case, the similar stabilization problems should be reconsidered, and some new problems for SMJSs will emerge. Based on these facts, a question whether there is a controller only in the diffusion section stabilizing a SMJS is proposed naturally. To the best of the authors' knowledge, such problems have not been fully investigated and still remain challenging, which need further more investigations.

In this paper, the stochastic stabilization problem of SMJSs via a stochastic controller only in the diffusion part is considered. The main contributions of this paper are as follows: (1) A new kind of stochastic controller is proposed such that the closed-loop system has a unique solution on $[0,\infty)$ and is almost surely exponentially admissible. Compared with some existing results, there is no additional assumption on system matrices; (2) New sufficient condition for the solvability of the desired controller is given in a set of LMIs, which could be solved easily and directly; (3) More general cases such as TPM partially unknown and system states partially observable are considered, and all the control gains are also obtained by solving some LMIs.

Notation: $\mathbb{R}^n$ denotes the n dimensional Euclidean space, $\mathbb{R}^{m\times n}$ is the set of all $m\times n$ real matrices. $\lambda_{\max}(M)$ and $\lambda_{\min}(M)$ denote the maximum and minimum value of matrix M respectively. In symmetric block matrices, we use $*$ as an ellipsis for the terms induced by symmetry.

2. Problem formulation

Consider a class of SSMJSs described as

$$E\mathrm{d}x(t) = A(\eta_t)x(t)\mathrm{d}t + u_\omega(t)\mathrm{d}\omega(t) \tag{1}$$

where $x(t)\in\mathbb{R}^n$ is the state vector, $u_\omega(t)$ is control input in the diffusion part, $\omega(t)$ is a one-dimensional Brownian motion or Wiener process. The underlying complete probability space is $(\Omega,\mathcal{F},\mathcal{F}_t,\mathbb{P})$ with a filtration $\{\mathcal{F}_t\}_{t\geqslant 0}$ satisfying the usual conditions (i.e. it is right continuous and $\mathcal{F}_0$ contains the $\mathbb{P}$-null sets). Matrix $E\in\mathbb{R}^{n\times n}$ may be singular, which is assumed to be $\mathrm{rank}(E)=r\leqslant n$. Matrix $A(\eta_t)$ is a known matrix of compatible dimension. $\{\eta_t, t\geqslant 0\}$ is a stationary ergodic Markov process and takes values in a finite set $\mathbb{S}\triangleq\{1,\ldots,N\}$, with TRM $\Lambda\triangleq(\lambda_{ij})\in\mathbb{R}^{N\times N}$ given by

$$\Pr\{\eta_{t+h}=j|\eta_t=i\} = \begin{cases}\lambda_{ij}h+o(h) & i\neq j\\ 1+\lambda_{ii}h+o(h) & i=j\end{cases} \tag{2}$$

where $h>0, \lim_{h\to 0^+}(o(h)/h)=0$, and $\lambda_{ij}\geq 0$, if $i\neq j, \lambda_{ii}=-\sum_{j\in\mathbb{S}}\lambda_{ij}$. For such a Markov process, it is obtained that

$$\sum_{j\in\mathbb{S}}\lambda_{\infty j}=1,\quad \lambda_{\infty j}>0 \tag{3}$$

where $\lambda_{\infty j}$ is the jth element of vector $\lambda_\infty\triangleq[\,\lambda_{\infty 1}\quad\lambda_{\infty 2}\quad\cdots\quad\lambda_{\infty N}\,]$ named as the steady state distribution of stationary(or transition probability matrix). The computation of λ_∞ can be solved by $\lambda_\infty=\hat{e}(\Lambda+\mathbb{E})^{-1}$, where $\hat{e}=[1\quad 1\quad\cdots\quad 1]$, $\mathbb{E}=\left[\hat{e}^T\quad\hat{e}^T\quad\cdots\quad\hat{e}^T\right]$. The detailed process is presented as follow. Because λ_∞ is the steady state distribution of stationary ergodic Markov process η_t, its element $\lambda_{\infty j}$ satisfies

$$\sum_{i\in\mathbb{S}}\lambda_{ij}\lambda_{\infty i}=0 \tag{4}$$

Based on (3) and (4), one has

$$\lambda_\infty(\Lambda_i+\hat{e}^T)=1 \tag{5}$$