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Finite element method with nonlocal boundary condition for solving the nondestructive testing problem of wood moisture content ☆

Jue Wang [a,b,*], Lei Zhang [c], FuMing Ma [a]

[a] School of Mathematics, Jilin University, Changchun 130012, China
[b] School of Mathematics, Harbin Engineering University, Harbin 150001, China
[c] School of Mathematics, Heilongjiang University, Harbin 150080, China

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ABSTRACT

In this paper, we use the finite element method (FEM) with nonlocal boundary condition for solving the nondestructive testing problem of wood moisture content based on a planar capacitance sensor model (i.e. the DtN-FEM for solving the mathematical model which is described by the exterior problems of a class of 3D Laplace equation with complicated boundary conditions). The original boundary value problem is reduced to an equivalent nonlocal boundary value problem via a Dirichlet-to-Neumann (DtN) mapping represented in terms of the Fourier expansion series. For numerical computation, a series of approximate problems with higher accuracy can be derived if one truncates the series term in the variational formulation, which is equivalent to the reduced problem. The error estimate is presented to show how the error depends on the finite element discretization and the accuracy of the approximate problem. Based on the numerical results, the relationship between the dielectric constant (DC) of tested wood and the capacitance value of the sensor is discussed. Finally, we applied a least squares fitting method (LSFM) to reconstruct the wood moisture content (WMC) from the data measured with a planar capacitance sensor. Compared with popular statistical methods, the hybrid experimental–computational method is more convenient and faster, and a large number of experiments are avoided, the costs of testing are reduced.

1. Introduction

The dielectric property of materials has been widely used in engineering and natural sciences. The earliest literature can be traced back to the year 1907 in Nature. Capacitive sensors is one of the most important applications of the dielectric property, they can directly sense a variety of things motion, electric field, chemical composition, and indirectly sense many other variables which can be converted into motion or dielectric constant, such as pressure, acceleration, and fluid composition. Single planar capacitance sensor is one of the high-precision equipments (see Fig. 1), which can be used for non-conductive humidity testing of a solid dielectric [see e.g. 1–3]. It is a special case of capacitance sensors with arbitrary shape.

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* Corresponding author at: School of Mathematics, Harbin Engineering University, Harbin 150001, China.
E-mail addresses: wangjue3721@163.com (J. Wang), zl19802003@163.com (L. Zhang), mfm@jlu.edu.cn (F. Ma).

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We used two-piece sheet metal as two poles of the detection capacitor, immobilized at two places, i.e., Ω_1 and Ω_2, where Ω is the current location of the tested material.

Partial differential equations (PDE) theory is a quantum model that has proved very successful in real applications [see e.g. 4]. We had established the precise mathematical model for the capacitance sensor with arbitrarily shape in [5,6], which is described by 3D Laplace equation with complex boundary conditions on the unbounded domain

$$\begin{cases} \nabla \cdot (\beta \nabla V) = 0,\ \mathbf{x} = (x_1, x_2, x_3) \in \Omega \bigcup \left(\mathbb{R}^3 \setminus (\overline{\Omega}_1 \cup \overline{\Omega}_2 \cup \overline{\Omega})\right), & \text{(1a)} \\ V|_{\overline{\Omega}_1} = V_1,\ V|_{\overline{\Omega}_2} = V_2, & \text{(1b)} \\ V|_{\partial\Omega_{in}} = V|_{\partial\Omega_{out}}, & \text{(1c)} \\ \varepsilon \frac{\partial V}{\partial n}\Big|_{\partial\Omega_{in}} = \tilde{\varepsilon} \frac{\partial V}{\partial n}\Big|_{\partial\Omega_{out}}, & \text{(1d)} \\ V \to 0,\ as\ |\mathbf{x}| = \sqrt{x_1^2 + x_2^2 + x_3^2} \to \infty. & \text{(1e)} \end{cases}$$

where $V(\mathbf{x}) = V(x_1, x_2, x_3)$ is the potential distribution function, $\frac{\partial}{\partial n}$ is the exterior normal derivative. $\Omega_1, \Omega_2, \Omega \subset \mathbb{R}^3$ are bounded open set, and satisfy $\overline{\Omega}_1 \bigcap \overline{\Omega}_2 \bigcap \overline{\Omega} = 0$. Ω is the spatial region of the tested material. Ω_1 and Ω_2 are the spatial region of the two poles of the capacitance sensor respectively. $\beta|_{\Omega} = \varepsilon,\ \beta|_{\mathbb{R}^3 \setminus (\overline{\Omega}_1 \cup \overline{\Omega}_2 \cup \overline{\Omega})} = \tilde{\varepsilon}$, it is a subfield constant, where ε is the dielectric constant of tested material. $\tilde{\varepsilon}$ is the dielectric constant of medium. $\partial\Omega_{in}$ and $\partial\Omega_{out}$ are respectively inside and outside of $\partial\Omega$. V_1 and V_2 denote respectively the potential on $\overline{\Omega}_1$ and $\overline{\Omega}_2$. Let Γ_1 denotes boundary $\partial\Omega_1$ of Ω_1, Γ_2 denotes boundary $\partial\Omega_2$ of Ω_2.

Clearly, problem (1) is the exterior problem, for numerical simulations, it is required a truncation of the computational domain to one of the finite extent. The reduced problem should be well-posed, convenient to implement, and yield accurate approximations to solutions of the original problem, so numerical approximation to the solutions of 3D Laplace equation in unbounded domain has attracted much attentions of engineers and mathematicians. Many effective and efficient methods (such as boundary integral equation method (BIEM), finite element method (FEM), etc.) had been proposed for different problems arising from various research areas. Krutitskii [7], Ahner and Howard [8] considered the BIEM for the exterior problem of Laplace equation. Although BIEM has many advantages to deal with the unbounded domain problem, the FEM is another important method to deal with this problem [see e.g. 9]. When using the finite element methods, an important task is to set up an artificial boundary condition to limit the computational domain, which is usually called DtN mapping or DtN artificial boundary condition. Han and Xu [10], Han et al. [11], Han and Zheng [12] designed various types of artificial boundary conditions to solve the exterior problem of some elliptic equations. In the paper by Givoli and Keller [13], they presented the DtN artificial boundary condition for the exterior problem of Laplace equation in three-dimension. For more information on this approach, the reader is referred to the review papers by Givoli [14] and Tsynkov [15]. In this paper, we concentrate on the model of a class of capacitance sensor, i.e. we will solve 3D Laplace equation with complex boundary conditions on the unbounded domain with the above ideas. Based on the variational principal, we proved that the modeling problem is well posed. Meanwhile, we analyzed the theoretical and numerical results of the finite element method for the problem (1).

This paper is organized as follows. In Section 2, we designed an exact artificial boundary condition on the spherical surface. In Section 3, the equivalence variational form of modeling problem and its approximation is derived. We proved the modeling problem and its approximation are well posed, and the approximate variational problem convergence to the original problem. In Section 4, the error analysis for finite element discretization is presented. In Section 5, the numerical examples are presented to demonstrate the performance of the proposed method, and the relationship between the dielectric constant of wood and the capacitance value of the sensor is discussed. This paper concludes in Section 6.

2. The artificial boundary condition on the spherical surface Γ_R

In this section, we will first derive the accurate artificial boundary condition and the approximate artificial boundary condition on the spherical surface artificial boundary Γ_R . Let $\tilde{\varepsilon} = 1$ (vacuum and atmosphere status). We will consider the 3D exterior problem of the following partial differential equations

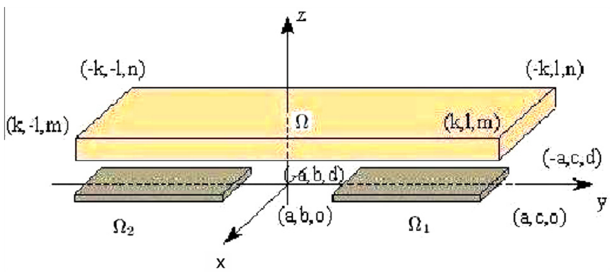

Fig. 1. Schematic diagram of single planar capacitive sensor.