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# Simulating the nonlinear acoustic oscillations in a resonator by gas-kinetic scheme

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ABSTRACT

The nonlinear oscillation in a compressible air-filled two-dimensional cylindrical resonator driven by a loudspeaker is simulated by using the gas-kinetic scheme. The influences of shock wave and higher harmonic on the time and space distribution of acoustic variables are investigated numerically for the practical applications of high-intensity acoustic devices. The validation of the developed model is verified by comparing the numerical results of pressure distribution with the theoretical ones for the finite-amplitude case. And then, the verified gas-kinetic scheme is used to simulate the acoustic field of highly nonlinear standing wave. Some interesting physical phenomena have been revealed for the highly nonlinear case. Sharp velocity spikes accompanied by the saw-tooth pressure waveforms appear at the end of the resonator. Moreover, the pressure at the position of theoretical pressure node is not zero and its frequency is about twice of the resonance frequency. Furthermore, the second harmonic is predominant at the location of pressure node. And nonlinear saturation can be found in tandem as the shock wave appears. Additionally, quasi-one-dimensional distribution accompanied changing flow direction and annular effect is observed for the spatial distribution of *x*-velocity. In addition, the *y*-velocity is in an irregular two-dimensional distribution and the *y*-velocity is not any more negligible relative to the *x*-velocity. Meanwhile, the important impacts as well as the causes of these nonlinear phenomena are analyzed. The results demonstrate the gas-kinetic scheme is an efficient and appropriate method for simulation of highly nonlinear acoustic oscillation and concerned problems.



## 1. Introduction

The nonlinear acoustic standing waves can be categorized in two groups [1]: finite-amplitude nonlinear standing wave and high-amplitude nonlinear standing wave. The study of nonlinear acoustic standing waves in closed tubes is very important in designing a wide range of systems, such as thermoacoustic devices [2,3], acoustic compressor [4,5], high quality resonators [6]. A large numbers of studies dealing with the analytical [7–9], numerical [10–12,14] and experimental studies [15,16] of the finite-amplitude nonlinear standing waves can be found in the literature.

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Analytical models [7–9] assumed that within a narrow resonance band the disturbances in the thermodynamic variables are of the order of $O(\varepsilon)$, where $\varepsilon = (\pi\, l/L)^{1/2}$ is a small parameter, $l$ and $L$ are the driving amplitude and tube length, respectively. Experimental evidence [16] showed that there was a substantial discrepancy between the analytical results and the experimental measurements when a high pressure amplitude sound field was used. Ilgamov et al. [17] reviewed many theoretical and experimental studies and found that the discrepancy between theory and experiments increases with $\varepsilon$. They concluded that all existing theories are unsatisfactory when $\varepsilon \geqslant 0.1$. And, when nonlinear terms in the conservation equations are retained, great mathematical difficulty is encountered and the analytical solution even in special cases is very difficult or nearly impossible. Therefore, numerical approaches are frequently used.

Several studies dealing with the numerical solution of the high-amplitude nonlinear acoustic standing waves can be also found in the literature. Aganin et al. [18] made a comparison of solutions of the gas oscillation in a closed tube for two ideal gas models, the entropy conservation model and the total energy model, with finite difference method for $l/L = 1.61 \times 10^{-3}$ and $1.61 \times 10^{-2}$. They found that the use of entropy conservation model can obtain steady solutions rapidly for high - amplitude gas oscillation with $l/L = 1.61 \times 10^{-2}$. However, the influence of entropy changes on the course of development of longitudinal gas oscillations was considered in their studies only in one dimension. Alexeev and Gutfinger [19] investigated two-dimensional gas flow in closed resonance tubes experimentally and numerically with $l/L \geqslant 5.97 \times 10^{-3}$. They solved Navier–Stokes equations and turbulence model with finite-difference algorithm and verified the model by comparison with experimental data. Their model, however, required extensive computational resources and time. A study of nonlinear acoustic waves in homentropic, i.e. uniform and constant entropy, perfect gas was presented by Christov et al. [20]. They solved the unsteady nonlinear wave equation using a Godunov-type shock-capturing scheme for $l/L = 8.44 \times 10^{-2}$, their numerical scheme did a very good job of capturing the "shocking-up" of the profiles as the time of blow-up approached. They however, did not consider the effect of thermoviscous attenuation. Nabavi [21] studied highly nonlinear standing wave oscillation in a thermoviscous fluid numerically by combination a fourth-order compact finite difference scheme and a fourth-order Runge–Kutta time stepping scheme. The result is an accurate and fast-solver numerical model which can predict the pressure, particle velocity and density along the high-amplitude nonlinear standing wave resonator filled with a thermoviscous fluid with no restriction on nonlinearity level and type of fluid. However, when $l/L > 9.59 \times 10^{-3}$, there are no reliable numerical experiments for air in their studies. Meanwhile, their model is a one-dimension model.

Due to the appearance of shock waves in resonant oscillations and the extremely small Mach number of the gas flow, a high-resolution numerical scheme and enormous computation resources as well as time are required [22]. The existing macroscopic numerical methods would face some numerical challenges to different degree, especially for two- or three-dimensional model taking into account viscous and heat conduction effects. Therefore, it is necessary to explore more appropriate and efficient numerical schemes. Ref. [23] presented two-dimensional simulated results using lattice Boltzmann method (LBM) on finite-amplitude nonlinear acoustic oscillations in a resonant tube with $l/L = 1.86 \times 10^{-3}$. But, the ability of LBM in compressible fluid as well as high-amplitude nonlinear acoustic oscillation problems is still limited [24]. By contrary, gas-kinetic scheme possesses significantly advantages in the compressible flow and highly nonlinear acoustic oscillation problems. Tang et al. [22] simulated two-dimension resonant oscillations by using a kinetic flux-vector splitting scheme (KFVS) to compute the fluxes with $l/L = 1.86 \times 10^{-3}$. Weak shock waves propagating inside the tube at the resonant frequency and slightly off-resonance frequencies are numerically captured. However, their numerical model did not consider highly nonlinear case. Feng et al. [25] simulated the nonlinear acoustic streaming motion in a compressible air-filled two-dimensional cylindrical resonator by using the gas-kinetic scheme, five cases with different excitation amplitudes are considered in simulation ranging from the linear to highly nonlinear regions ($l/L$ from $1.15 \times 10^{-5}$ to $5.76 \times 10^{-3}$), the results demonstrated that the gas-kinetic scheme is capable of resolving large Reynolds number of nonlinear acoustic problems.

In the present manuscript, the gas-kinetic scheme is used to simulate the high-amplitude nonlinear acoustic oscillation in closed resonator. Unlike the previous investigation [25], the physical phenomena in high-amplitude nonlinear standing wave and the causes for these nonlinear phenomena are the main focus of the present work. The results of the present investigation may be used in the design of various high-intensity acoustic devices.

The manuscript is organized as follows. In Sections 2 and 3, the detailed descriptions of the gas-kinetic scheme and the models are given. In Section 4, the results for finite-amplitude nonlinear standing wave with $l/L = 1.86 \times 10^{-3}$ were compared with theoretical analysis and experimental data first. And then, the verified gas-kinetic scheme is used to simulate the acoustic field of high-amplitude nonlinear standing wave with $l/L = 1.86 \times 10^{-2}$, some interesting physical phenomena are revealed, and detailed explanations are provided for these phenomena. Finally, a brief conclusion of the present study is given in Section 5.

## 2. Gas-kinetic scheme

The present two-dimensional gas-kinetic scheme is similar to Xu's method [26,27]. The main idea of this scheme will be briefly described in this section. Interested readers can refer Refs. [26,27] for more details about this scheme. The gas-kinetic scheme is a finite-volume method, which obtains the numerical flux from the time-dependent gas distribution function.

The present gas-kinetic BGK (GKS-BGK) scheme is based on the two-dimensional BGK-Boltzmann equation

$$f_t + uf_x + vf_y = \frac{g - f}{\tau}, \tag{2.1}$$