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# On the computation of inverses and determinants of a kind of special matrices ☆

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ABSTRACT

In this paper, the inverse and determinant of a special kind of centrosymmetric matrices are investigated. Based on the partition property of a matrix with centrosymmetric structure and algorithms for the inverse and determinant proposed in Chen and Yu (2011), a computation algorithm for the inverse and determinant of a centrosymmetric matrix is finally developed.



## 1. Introduction and preliminaries

For convenience, let us first recall some standard notation and definitions.

$C^{m\times n}$ and $R^{m\times m}$ denote the $n$-dimensional complex and real matrix space, respectively, and $I_n$ stands for the unit matrix with order $n$.

Centrosymmetric matrix is one of the important matrices in many application such as in the numerical analysis, the theory of control, digital signal image processing, and could be applied to the mathematical representation of high dimensional, nonlinear electromagnetic interference signals (Toplitz matrices, for example, is a special kind of centrosymmetric matrix). A lot of work has been done on centrosymmetric matrices (see, e.g., [2–8]). In this article, we investigate the inverse and determinant of a special kind of centrosymmetric matrix. Such matrices occur in many problems.

For this paper, we will use the following definition.

**Definition 1.1.** $A=(a_{ij})_{n\times n}\in R^{n\times n}$ is a centrosymmetric matrix, if

$$a_{ij}=a_{n-i+1,n-j+1}, 1\leqslant i\leqslant n, 1\leqslant j\leqslant n, \quad \text{or equivalently}, \quad J_nAJ_n=A,$$

where $J_n=(e_n,e_{n-1},\ldots,e_1)$, and $e_i$ is the unit vector with the $i$-th elements 1 and others 0.

We mainly discuss the case when $n$ is even. The structure of a centrosymmetric matrix can be exploited by the following lemma (see, e.g., [2,4–6]).

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**Lemma 1.1** [2]. *Let* $A = (a_{ij})_{n\times n} \in R^{n\times n} (n = 2m)$. *Then A is centrosymmetric, if and only if A has the form*

$$A = \begin{pmatrix} B & J_m C J_m \\ C & J_m B J_m \end{pmatrix}, \quad \text{and} \quad Q^T A Q = \begin{pmatrix} B - J_m C & 0 \\ 0 & B + J_m C \end{pmatrix}, \tag{1.1}$$

*where* $B \in R^{m\times m}, C \in R^{m\times m}$, *and* $Q = \frac{\sqrt{2}}{2}\begin{pmatrix} I_m & I_m \\ -J_m & J_m \end{pmatrix}$.

For further discussion, we will introduce the Hessenberg matrix.

**Definition 1.2** [1]. An $m \times m$ matrix $A = (a_{ij})_{m\times m}$ is called a lower Hessenberg matrix, if and only if $A$ has the following form

$$A = \begin{pmatrix} a_{11} & a_{12} & & & \\ a_{21} & a_{22} & a_{23} & & \\ \vdots & \vdots & \ddots & \ddots & \\ a_{m-1,1} & a_{m-1,2} & \cdots & a_{m-1,m-1} & a_{m-1,m} \\ a_{m,1} & a_{m,2} & \cdots & a_{m,m-1} & a_{m,m} \end{pmatrix}. \tag{1.2}$$

Without loss of generality, we assume that $A$ is not singular, and all elements of the super diagonal of $A$ are non-zero, i.e., $a_{i,i+1} \neq 0$ for $i = 1, 2, \ldots, m-1$. There are some useful properties on the inverse and determinant of the Hessenberg matrix $A$.

Based on A, we can construct an $(m+1) \times (m+1)$ lower triangular matrix

$$\tilde{A} = \left(\begin{array}{c|c} e_1^T & 0 \\ \hline A & e_m \end{array}\right) = \left(\begin{array}{ccccc|c} 1 & 0 & \cdots & \cdots & 0 & 0 \\ \hline a_{11} & a_{12} & & & & 0 \\ a_{21} & a_{22} & a_{23} & & & 0 \\ \vdots & \vdots & \ddots & \ddots & & 0 \\ a_{m-1,1} & a_{m-1,2} & \cdots & a_{m-1,m-1} & a_{m-1,1} & 0 \\ a_{m,1} & a_{m,2} & \cdots & a_{m,m-1} & a_{m,1} & 1 \end{array}\right), \tag{1.3}$$

where $e_1$ and $e_m$ are the first and the last column of matrix $I_m$, respectively. It is obvious that $\tilde{A}$ is non-singular. Thus, we can assume that $\tilde{A}^{-1} = \begin{pmatrix} \alpha & L \\ h & \beta^T \end{pmatrix}$, where $\alpha, \beta, h$ are $m$-dimensional vectors, and $L$ is an $m \times m$ matrix. Then, we have the following lemmas on the inverse and determinant of $A$.

**Lemma 1.2** [1]. *Let* $A \in R^{m\times m}$ *be a Hessenberg matrix, and the lower triangular matrix* $\tilde{A}$, *and* $\alpha, \beta, h, L$ *are defined aforementioned. Then*

(1) $A^{-1} = L - h^{-1}\alpha\beta^T$.
(2) $\det(A) = (-1)^m h \cdot \det(\tilde{A}) = (-1)^m h \cdot \prod_{i=1}^{m-1} a_{i,j+1}$.

**Lemma 1.3** [1]. *Let* $A \in R^{m\times m}$ *be a Hessenberg matrix, and* $\tilde{A}$ *is the corresponding lower triangular matrix aforementioned. Assume that* $\tilde{A}^{-1} = (c_1, c_2, \ldots, c_{m+1})$, *then all* $c_j$ *can be calculated recursively as follows:*

$$\begin{cases} c_{m+1} = e_{m+1}, \\ c_j = \left(ej - \sum_{i=j}^{m} a_{i,j} c_{i+1}\right) / a_{j-1,j}, \quad \text{for } j = m, m-1, \ldots, 1. \end{cases}$$

## 2. Main results

In this section, the inverse and determinant of the following $n$-by-$n$ $(n = 2m)$ centrosymmetric matrix are mainly considered: