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Numerical solution of linear Fredholm integral equations via two-dimensional modification of hat functions

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ABSTRACT

In this work we approximate the solution of the linear Fredholm integral equations, by means of a new two-dimensional modification of hat functions (2D-MHFs) and a new operational matrix of integration. By this idea, the basic equations will be changed into the associated systems of algebraic equations. Also, an error analysis is provided under several mild conditions. The method is computationally attractive and some numerical examples are provided to illustrate its high accuracy.

1. Introduction

Integral equations have gained a lot of interest in many applications, such as biological, physical, and engineering problems. The numerical methods for solution of Fredholm integro-differential equations have been investigated in many studies. Many problems in engineering and mechanics can be transformed into two-dimensional Fredholm integral equations of the second kind. For example, it is usually required to solve Fredholm integral equations in the calculation of plasma physics [1]. There are many works on developing and analyzing numerical methods for solving Fredholm integral equations of the second kind (see [2–10]).

The methods heretofore available which can solve high dimensional equations are the radial basis functions (RBFs) method [11,12], the spline functions method [13], the block pulse functions (BPFs) method [14], the spectral methods such as collocation and Tau method [9,10,15–17], Nystroms method [4,7], transform methods [18], Adomian decomposition method (ADM) [19–21], wavelets methods [22] and many other methods [23–25].

In the present paper, we use 2D-MHFs to solve the linear equation

$$f - \mathcal{K}f = g, \tag{1}$$

where $\mathcal{K}$ is a compact linear operator on the Banach space $\mathcal{X}$. The operator $(I - \mathcal{K})$ is assumed to be invertible, so that the equation has a unique solution $f \in \mathcal{X}$ for any given $g \in \mathcal{X}$. Let $\mathcal{K}$ be the compact linear integral operator defined by

$$\mathcal{K}f(x,y) = \int_0^1 \int_0^1 k(x,y,s,t)f(s,t)dtds, \quad (x,y) \in D = [0,1] \times [0,1],$$

where $\mathcal{X} = C^4(D)$ and the kernel function $k \in L^2(D \times D)$. A standard technique to solve (1) approximately is to replace $\mathcal{K}$ by a finite rank operator. The approximate solution is then obtained by solving a system of linear equations. For the integral Eq. (1), consider the iteration

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$$f^{(n+1)} = g + \mathcal{K}f^{(n)}, \quad n = 0, 1, \ldots .$$

From the geometric series theorem, it can be shown that this iteration converges to the solution f if $||\mathcal{K}||_\infty < 1$, and in that case

$$||f - f^{(n+1)}||_\infty \leqslant ||\mathcal{K}||_\infty ||f - f^{(n)}||_\infty.$$

Similarly Sloan [26], we can show that one such iteration is always a good idea if the initial guess is the solution obtained by the Galerkin method, regardless of the size of $\mathcal{K}$.

2. Definitions of one-dimensional modification of hat functions

In an $(m+1)$-set of one-dimensional modification of hat functions (1D-MHFs) over interval $[0, 1]$, are defined as:

$$h_0(x) = \begin{cases} \frac{1}{2h^2}(x-h)(x-2h) & 0 \leqslant x \leqslant 2h, \\ 0 & \text{otherwise}, \end{cases}$$

if i is odd and $1 \leqslant i \leqslant m-1$,

$$h_i(x) = \begin{cases} \frac{-1}{h^2}(x-(i-1)h)(x-(i+1)h) & (i-1)h \leqslant x \leqslant (i+1)h, \\ 0 & \text{otherwise}, \end{cases}$$

if i is even and $2 \leqslant i \leqslant m-2$,

$$h_i(x) = \begin{cases} \frac{1}{2h^2}(x-(i-1)h)(x-(i-2)h) & (i-2)h \leqslant x \leqslant ih, \\ \frac{1}{2h^2}(x-(i+1)h)(x-(i+2)h) & ih \leqslant x \leqslant (i+2)h, \\ 0 & \text{otherwise}, \end{cases}$$

and

$$h_m(x) = \begin{cases} \frac{1}{2h^2}(x-(1-h))(x-(1-2h)) & 1-2h \leqslant x \leqslant 1, \\ 0 & \text{otherwise}, \end{cases}$$

where $m \geqslant 2$ is an even integer and $h = \frac{1}{m}$. It is obvious that

$$h_i(jh) = \begin{cases} 1 & i = j, \\ 0 & i \neq j, \end{cases}$$

and

$$\sum_{i=0}^{m} h_i(x) = 1.$$

Let us write the 1D-MHFs vector $H(x)$ as follows:

$$H(x) = [h_0(x), h_1(x), \ldots, h_m(x)]^T; \quad x \in [0, 1].$$

Simple calculation show that:

$$\int_0^1 H(x)H^T(x)dx = p, \tag{2}$$

where p is the $(m+1) \times (m+1)$ matrix as follows

$$p = \frac{h}{15}\begin{pmatrix} 4 & 2 & -1 & & & & & & & \\ 2 & 16 & 2 & 0 & & & & & & \\ -1 & 2 & 8 & 2 & -1 & & & & & \\ & \ddots & \ddots & \ddots & \ddots & \ddots & & & & \\ & & \ddots & \ddots & \ddots & \ddots & \ddots & & & \\ & & & \ddots & \ddots & \ddots & \ddots & \ddots & & \\ & & & & \ddots & \ddots & \ddots & \ddots & \ddots & \\ & & & & & -1 & 2 & 8 & 2 & -1 \\ & & & & & & 0 & 2 & 16 & 2 \\ & & & & & & & -1 & 2 & 4 \end{pmatrix}. \tag{3}$$