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Impact of keeping silence on spatial reciprocity in spatial games

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ABSTRACT

In social systems, the purpose of individuals playing games is to get higher payoffs. However, if the benefit from game interactions does not achieve their expectation, agents may be more inclined to escape from games to reduce the potential consumption. Of particular interest, this trait could be mimicked by the so-called "silence" strategy. In this work, we consider silence strategy in the framework of prisoner's dilemma game, where players either engage in the game as cooperators or defectors, or gain no any payoff as the silence agents. The events of turning into and escaping from silence strategy depend on both the consumption level and silence period. Of particular interest, it is unveiled that there exists an intermediate consumption level that could guarantee the optimal cooperation circumstance. For the small consumption level, the silence strategy could enhance the frequency of cooperation through the rock–scissor–paper cycle. While for the large consumption level, vast majority of players choose the silence strategy to avoid the high loss of engaging in games. This discovery is universally effective for the silence period.

1. Introduction

The emergence and maintenance of cooperation among self-interested individuals is an open challenge of social dilemmas in which individual, local interests are inconsistent with collective, global benefits. Study of evolutionary games, borrowing from the technology from biology, economy, computer sciences, physics and sociology, has provided powerful support to explore this problem [1–5]. Typical examples include prisoner's dilemma game (PDG) [6–10], snow-drift game (SG) [11–13] and public goods game (PGG) [14–17]. While among them, prisoner's dilemma game (PDG) attracts the great attention form both experiment and theoretical viewpoints. In its basic version, two players simultaneously decide to take one strategy: cooperation (C) or defection (D). Both of them can receive the reward R upon mutual cooperation and the punishment P upon mutual defection. However, if one defects while the other cooperates, the defector gets the temptation T and the cooperator is left with the sucker's payoff S. These payoffs satisfy the ranking $T > R > P > S$ and $2R > T + S$. Thus, defection optimizes the individual payoff, in spite of the fact that mutual cooperation could yield a higher collective benefit. This theoretical prediction seems inconsistent with the ubiquitous observation of cooperation behaviors in our realistic life.

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Through the development of several decades, the spatial structured population has been proved to be a very useful framework, in which the cooperators can survive by forming clusters to resist the invasion of defectors [18]. After this seminal discovery, the role of spatial structure and its various underlying promoting mechanisms in evolutionary games, have been intensively explored [19–21]. Typical examples include the heterogeneous activities [22–24], reputation [25,26], punishment [27–29] and reward [30], migration [31], conditional strategy [32,33] and sharing the gains [34], to name but a few. Among them, there is one particular scenario, voluntary participation [35–37], attracting much attention. Under this setup, the loner, as the novel strategy, does not engage in any interaction yet obtains a fixed and small payoff. For example, in [36] the authors found that the voluntary participation could effectively prevent the tragedy of the commons [38].

However, in the realistic systems, we usually observe another type of phenomenon: once the player does not engage in the games, it can not gain any payoff from its neighbors again. While the player will keep this trait for a certain period rather than one step. After this period, it can return to the normal player. Combining this fact and the suggestion of the lowest payoff for the individual survival [39], it is natural to take into account a new scenario: if the player who has less payoff may quit the games, how does this affect the evolution of cooperation? Inspired by this question, we consider a new strategy, silence, in the prisoner's dilemma game, where player with low payoff is likely to choose this behavior for a particular period. While for the criterion of turning into the silence strategy, we introduce the consumption level. By using the systematic simulations, we find that the introduction of silence strategy can enhance the density of cooperation for low consumption level, while impedes the evolution of cooperation at high consumption level. This observation is robust against the silence period.

2. Model

The prisoner's dilemma game (PDG) is implemented on the $M \times M$ square lattice with the von Neumann neighborhood ($k = 4$) and period boundary condition. Initially, each node i is designated either as a cooperator ($s_i = C$) or defector (D) with equal probability. At each time step, individual i can acquire its accumulated payoff P_i by playing the game with all its neighbors. For simplicity, but without loss of generality, the payoffs can be rescaled such that we set $T = b$, $R = 1$ and $P = S = 0$, where $1 \geqslant b \geqslant 2$ ensures a proper payoff ranking and preserves the essential dilemma between individual profits and welfare of the population for repeated games.

Motivated by previous study of cascading bankruptcy process, where the survival payoff of a player is proportional to the normal payoff in the state of all the players choosing cooperators [35], we define a critical payoff to quantify individual probability of selecting the novel strategy silence:

$$T_c = (b - 1)k, \tag{1}$$

Herein T_c denotes the extra payoff when player i chooses defection and cooperation (in the circumstance that all its neighbors simultaneously adopt cooperation), respectively. In the same way, its surplus payoff is defined as follows,

$$S_i = (1 - \eta)P_i, \tag{2}$$

where the term $\eta * P_i$ denotes the consumption of player during the game. Once the extra payoff T_c and surplus payoff S_i are known at time step t, player i chooses the strategy silence at the $t + 1$ time step with the following probability,

$$p_s(i) = \frac{1}{1 + e^{[(S_i - T_c)/K]}}, \tag{3}$$

where K quantifies uncertainty of strategy adoption. To be simple, we simply fix the value of K to be $K = 0.1$ in the present paper. Obviously, $\eta = 0$ in Eq. (2) means that player i has enough surplus payoff to participate in the games and can afford the consumption, which causes the selection probability of silence p_s to seem negligible. At variance, the case of $\eta = 1$ corresponds to no surplus payoff, which means that player must choose silence to reduce the consumption. Once player i choose silence, it can not engage in any game (namely, no payoff) in the following L steps. Interestingly, this non-Markovian setup is similar to the freezing period in the study of opinion dynamics, which leads to the fastest consensus [40]. In addition, if the time exceeds the silence period L, player i will be re-designed either as a cooperator or defector with equal probability again.

Given that the player i does not fall into the silence period, it can update its strategy by randomly choosing one neighbor y and adopts the its strategy s_y in accordance with the probability [31]:

$$W(s_x \leftarrow s_y) = \frac{1}{1 + e^{[(P_{s_x} - P_{s_y})/K]}}. \tag{4}$$

It is worth mentioning that even if the neighbor y possesses silence strategy, it can also be adopted.

Simulations are performed on square networks ranging from $M = 100$ to $M = 300$ to avoid accidental extinction of the competing strategies. The density of strategies is recorded after the system reaches dynamical equilibrium, *i.e.*, the average density of strategies is determined within 10^5 steps after sufficiently long transients are discarded. Moreover, since silence period may introduce additional disturbances, the final results are averaged over up to 50 independent runs for each set of parameter values in order to assure suitable accuracy.