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A hybrid viscosity algorithm via modify the hybrid steepest descent method for solving the split variational inclusion in image reconstruction and fixed point problems

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ABSTRACT

In this paper, we introduce and study a new viscosity approximation method by modify the hybrid steepest descent method for finding a common solution of split variational inclusion problem and fixed point problem of a countable family of nonexpansive mappings. Under suitable conditions, we prove that the sequences generated by the proposed iterative method converge strongly to a common solution of the split variational inclusion problem and fixed point problem for a countable family of nonexpansive mappings which is the unique solution of the variational inequality problem. The results present in this paper are the supplement, extension and generalization of the previously known results in this area. Numerical results demonstrate the performance and convergence of our result that the algorithm converges to a solution to a concrete split variational inclusion problem and fixed point problem.

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1. Introduction and background

The problem of image reconstruction from projections can be represented by a system of linear equations

$$Ax = b. \tag{1.1}$$

We see that, the system (1.1) is often inconsistent, and one usually seeks a point which minimizes $x \in \mathbb{R}^n$ by some predetermined optimization criterion. The problem is frequently ill-posed and there may be more than one optimal solution. The standard approach to dealing with that problem is via regularization. The well-known *convex feasibility problem* (CFP); Suppose that $C_1, \ldots, C_N$ are finitely many closed convex subset of a Hilbert space H with $C := \cap_i C_i \neq \emptyset$. The convex feasibility problem is simply;

$$\text{find a point } x^* \in C. \tag{1.2}$$

A special case of the convex feasibility problem is the *split feasibility problem* (SFP).

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In 1994, the SFP was first introduced by Censor and Elfving [4], in finite-dimensional Hilbert spaces, for modeling inverse problems which arise from phase retrievals and in medical image reconstruction. Since then, the SFP has received much attention due to its applications in signal processing, image reconstruction, with particular progress in intensity-modulated radiation therapy (IMRT), approximation theory, control theory, biomedical engineering, communications, and geophysics. For examples, one can refer to [4,10,13,20] and related literatures.

It is found that the SFP can also applied to study intensity-modulated radiation therapy (IMRT) (see, e.g. [1]), beams of penetrating radiation are directed at the tumor lesion from external sources. A multileaf collimator (MLC) is used to split each beam into many beamlets with individually controllable intensities. There are two principal aspects of radiation teletherapy that call for computational modeling. The first is the calculation of the radiation dose absorbed in the irradiated tissue based on a given distribution of beamlet intensities. This dose calculation is a *forward problem*. The second aspect is the *inverse problem* of the first: to find a distribution of radiation intensities (radiation intensity map) deliverable by all beamlets, which would result in a clinically-acceptable dose distribution (i.e., such that the dose to each tissue would be within the desired *upper* and *lower bound*s, which are prescribed on the basis of medical diagnosis, knowledge and experience). To be of practical value, however, this radiation intensity map must be *implementable*, in a clinically acceptable form, on the available treatment machine. Therefore, in addition to the physical and biological parameters of the irradiated object, the relevant information about the capabilities and specifications of the available treatment machine (i.e., radiation source) should be taken into account.

In intensity modulated radiation therapy, beamlets of radiation with different intensities are transmitted into the body of patient. Each voxel within the patient will then absorb a certain dose of radiation from each beamlet. The goal of IMRT is to direct a sufficient dosage to those regions requiring the radiation, those that are designated planned target volumes (PTVs), while limiting the dosage received by the other regions, the so-called organs at risk (OAR). The forward problem is to calculate the radiation dose absorbed in the irradiated tissue based on a given distribution of the beamlet intensities. The inverse problem is to find a distribution of beamlet intensities, the radiation intensity map, which will result in a clinically acceptable dose distribution. One important constraint is that the radiation intensity map must be implementable; that is, it is physically possible to produce such an intensity map, given the machines design.

The equivalent uniform dose (EUD) for tumors is the biologically equivalent dose which, if given uniformly, will lead to the same cell kill within the tumor volume as the actual nonuniform dose. Constraints on the EUD received by each voxel of the body are described in dose space, the space of vectors whose entries are the doses received at each voxel. Constraints on the deliverable radiation intensities of the beamlets are best described in intensity space, the space of vectors whose entries are the intensity levels associated with each of the beamlets. The constraints in dose space will be upper bounds on the dosage received by the OAR and lower bounds on the dosage received by the PTV. The constraints in intensity space are limits on the complexity of the intensity map and on the delivery time, and, obviously, that the intensities be nonnegative. Because the constraints operate in two different domains, it is convenient to formulate the problem using these two domains. This leads to a split feasibility problem.

A number of image reconstruction problems can be formulated as the SFP; see, e.g. [10] and the reference therein. Recently, it is found that the SFP can also be applied to study intensity-modulated radiation therapy (IMRT); [4,13,14]. In the recent past, a wide variety of iterative methods have been used in signal processing and image reconstruction and for solving the SFP; see, e.g., [2,13,14,4,15–18,25,19] and the reference therein.

Image restoration and image reconstruction are the two main sub-branches of image recovery. The term image restoration usually applies to the problem of estimating the original form h of a degraded image x. Hence, in image restoration the data consist of measurements taken directly on the image to estimated, x being a blurred and noise-corrupted version of h. The blurring operation can be induced by the image transmission medium, e.g., the atmosphere in astronomy, or by the recording device, e.g., an out-of-focus or moving camera. On the other hand, image reconstruction refers to problems in which the data x are indirectly related to the form the original image h.

In this paper, we will present article, our main purpose is to study the split problem. First, we recall some background in the literature.

Problem 1 (*The split feasibility problem (SFP)*). Let C and Q be two nonempty closed convex subsets of Hilbert space H_1 and H_2, respectively and $A : H_1 \to H_2$ be a bounded linear operator. The *split feasibility problem* (SFP) is formulated as finding a point

$$x^* \in C \quad \text{such that } Ax^* \in Q, \tag{1.3}$$

which was first introduced by Censor and Elfving [4] in medical image reconstruction.

A special case of the SFP is the *convexly constrained linear inverse problem* (CLIP) in a finite dimensional real Hilbert space [5]:

$$\text{find } x^* \in C \text{ such that } Ax^* = b, \tag{1.4}$$

where C is a nonempty closed convex subset of a real Hilbert space H_1 and b is a given element of a real Hilbert space H_2, which has extensively been investigated by using the Landweber iterative method [6]:

$$x_{n+1} = x_n + \gamma A^T(b - Ax_n), \quad n \in \mathbb{N}.$$

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