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Existence and exponential stability of piecewise mean-square almost periodic solutions for impulsive stochastic Nicholson's blowflies model on time scales

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ABSTRACT

In this paper, a class of impulsive stochastic Nicholson's blowflies model with patch structure and nonlinear harvesting terms is introduced and studied on time scales. By using contraction mapping principal and Gronwall–Bellman inequality technique, some sufficient conditions for the existence and exponential stability of piecewise mean-square almost periodic solutions for the model with infinite delays are established on time scales. Finally, an example is given to demonstrate the validity of the conditions of the main theorem.

1. Introduction

On the applied aspect of dynamic systems, as every one knows, one of the most popular dynamic population models is Nicholson's blowflies model

$$N'(t) = -\delta N(t) + pN(t-\tau)e^{-aN(t-\tau)},$$

which was proposed by Gurney et al. [1] to describe the population of the Australian sheep-blowfly and to agree with Nicholson's experimental data [2]. Here, $N(t)$ is the size of the population at time t, p is the maximum per capita daily egg production, $1/a$ is the size at which the population reproduces at its maximum rate, δ is the per capita daily adult death rate, and τ is the generation time. The model and its modifications have been extensively and intensively studied and numerous results about its stability, persistence, attractivity, periodic solutions, almost periodic solutions and so on (see [3–7]) have been obtained.

In recent years, the theory of time scale, which was introduced by Hilger in his PhD thesis [8], has been established in order to unify continuous and discrete analysis. In fact, the progressive field of dynamic equations on time scales contains, links and extends the classical theory of differential and difference equations. This theory represents a powerful tool for applications to economics, biological models, quantum physics among others. See, for instance, Ref. [9]. Because of this fact, it has been attracting the attention of many mathematicians (see Refs. [10–14]).

On the other hand, recently, qualitative analysis of stochastic model has attracted the attention of many mathematicians and biologists due to the fact the natural extension of a deterministic model is stochastic model [15]. In the aspect of life sciences, most phenomena are basically modeled as suitable stochastic processes, where relevant parameters are modeled as suitable stochastic processes. Furthermore, the ecological systems are often characterized by the fact that they experience a sudden change of their state at certain moments. These systems are subject to short term perturbations which are usually

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described to be in the form of impulses in the modeling process. Impulsive effects widely exist in many evolution process of real-life sciences and real world applications such as automatic control, cellular neural networks, population dynamics [16–21]. Moreover, the theory of almost periodic stochastic process is an interesting issue for qualitative theory and the interest in this subject remains growing [22,23].

In fact, both continuous and discrete stochastic systems are very important in implementation and application. Therefore, the study of stochastic differential equations on time scales has received much attention, see [12,24], which displays a combination of characteristics of both continuous-time and discrete-time stochastic system. Also, stochastic differential equations with impulses provide an adequate mathematical model of many evolutionary processes that suddenly change their states at certain moments.

Furthermore, it is well-known that the optimal management of renewable resources has directly relationship to sustainable development of population. One way to handled this is to study population models subject to harvesting, dispersal or competition. Biologists have purported that the process of harvesting of population species, in particular, is of great significance in exploitation of biological resources such as in fishery, forestry and wildlife management (see Refs. [25,26]). Meanwhile, growth models given by patch-structured systems of delay differential equations (DDEs) have been recently studied by several authors, who analyzed the effect of dispersal in the local and global dynamics of the species, in terms of permanence in each patch, coexistence of different patches, local stability of equilibria, bifurcations, existence of global attractors, and several other features (see e.g. [27] and the references therein).

Motivated by the above, in this paper, we will be concerned with the following impulsive stochastic Nicholson's blowflies model with patch structure and nonlinear harvesting terms on time scales:

$$\begin{cases} \Delta x_i(t) = \Bigg[-\alpha_i(t)x_i(t) + \sum_{j=1}^{m}\Big(\beta_{ij}(t)x_j\big(t-\eta_{ij}(t)\big)e^{-\gamma_{ij}(t)\int_{-\infty}^{0}k_{ij}(s)x_j(t+s)\Delta s}\Big) \\ \qquad -H_i(t,x_i(t-\sigma_i(t)))\Bigg]\Delta t + \sum_{j=1}^{m}\delta_{ij}\big(t,x_j(t-\zeta_{ij}(t))\big)\Delta\omega_j(t), \quad t \neq t_k, \\ \widetilde{\Delta}x_i(t) = x(t_k+0) - x(t_k-0) = \alpha_{ik}x(t_k) + I_{ik}(x_i(t_k)) + v_{ik}, \quad t = t_k, \end{cases} \tag{1.1}$$

where $\Delta x_i(t)$ denotes a Δ-stochastic differential of $x_i(t)$, $\alpha_i, \beta_{ij}, \gamma_{ij}, \eta_{ij}, \sigma_i \in PC_{rd}(\mathbb{T},\mathbb{R}^+), H_i \in PC_{rd}(\mathbb{T}\times\mathbb{R},\mathbb{R}^+)$, $\{t_k\} \in \mathfrak{B}$, $\mathfrak{B} = \{\{t_k\} : t_k \in \mathbb{T},\, t_k < t_{k+1},\, k \in \mathbb{Z},\, \lim_{k\to\pm\infty} = \pm\infty\}$, the delay kernel $k_{ij} \in C\big((-\infty,0]_{\mathbb{T}},\mathbb{R}^+\big)$ and $\int_{-\infty}^{0}k_{ij}(t)\Delta t \leqslant \overline{k_{ij}}$, the constants $\alpha_{ik}, v_{ik} \in \mathbb{R}$ and $I_{ik} \in C(\mathbb{R},\mathbb{R}), \delta_{ij}$ is Borel measurable, $i = 1,2,\ldots,n,\ j = 1,2,\ldots,m,\ k \in \mathbb{Z}$ and $A = (\delta_{ij})_{n\times m}$ is a diffusion coefficient matrix. Here, in this model, $x_i(t)$ is the size of the population at time t in the ith unite, $\beta_{ij}(t)$ is the maximum per capita daily egg production at time t in the i unite, $1/\gamma_{ij}(t)$ is the size at which the population reproduces at its maximum rate at time t in the ith unite, $\alpha_i(t)$ is the per capita daily adult death rate at time t in the ith unite, $H := (H_1,H_2,\ldots,H_n)^T$ is the nonlinear harvesting term and $A = (\delta_{ij})_{n\times m}$ is the random perturbation term for the system. Let $(\Omega,\mathbb{F},\mathbb{P})$ be a complete probability space furnished with a complete family of right continuous increasing sub σ-algebras $\{\mathcal{F}_t : t \in [0,+\infty]_{\mathbb{T}}\}$ satisfying $\mathcal{F}_t \subset \mathbb{F}$. $\omega(t) = (\omega_1(t),\omega_2(t),\ldots,\omega_m(t))$ is an m-dimensional standard Brownian motion over $(\Omega,\mathbb{F},\mathbb{P})$. Some sufficient conditions are obtained ensuring the existence and exponential stability of mean-square almost periodic solutions for system (1.1) by fixed point theorem and Gronwall–Bellman inequality technique.

For convenience, we introduce the following notations:

$$\bar{f} = \sup_{t\in\mathbb{T}}|f(t)|, \quad \bar{g} = \sup_{(t,x)\in\mathbb{T}\times\mathbb{R}}|g(t,x)|,$$

where $f(t)$ is a mean-square almost periodic function and $g(t,x)$ is a mean-square uniformly almost periodic function on time scales, the concepts of which will be introduced in the next section, respectively.

2. Preliminaries

In this section, we present some basic concepts and results concerning time scales. For more details, the reader may want to consult Refs. [10,13].

A time scale $\mathbb{T}$ is a closed subset of $\mathbb{R}$. It follows that the jump operators $\sigma, \rho : \mathbb{T} \to \mathbb{T}$ defined by $\sigma(t) = \inf\{s \in \mathbb{T} : s > t\}$ and $\rho(t) = \sup\{s \in \mathbb{T} : s < t\}$ (supplemented by $\inf\phi := \sup\mathbb{T}$ and $\sup\phi := \inf\mathbb{T}$) are well defined. The point $t \in \mathbb{T}$ is left-dense, left-scattered, right-dense, right-scattered if $\rho(t) = t,\ \rho(t) < t,\ \sigma(t) = t,\ \sigma(t) > t$, respectively. If $\mathbb{T}$ has a right scatter minimum m, define $\mathbb{T}_k := \mathbb{T}\setminus m$; otherwise, set $\mathbb{T}^k = \mathbb{T}$. The notations $[a,b]_{\mathbb{T}},\ [a,b]_{\mathbb{T}}$ and so on, we will denote time scale intervals

$$[a,b]_{\mathbb{T}} = \{t \in \mathbb{T} : a \leqslant t \leqslant b\},$$

where $a,b \in \mathbb{T}$ with $a < \rho(b)$.

The graininess function is defined by $\mu : \mathbb{T} \to (0,\infty)$: $\mu(t) := \sigma(t) - t$, for all $t \in \mathbb{T}$.