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Existence and controllability results for second-order impulsive stochastic evolution systems with state-dependent delay ☆

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ABSTRACT

In this paper, we discuss a kind of impulsive second-order stochastic evolution systems with state-dependent delay in a real separable Hilbert space. The results concerning the existence and controllability of mild solutions have been addressed. By means of the fixed point techniques, some sufficient conditions are formulated, as well as an application involving partial differential equation with impulses is presented.

1. Introduction

The study of stochastic phenomena has been motivated by the necessity of taking into account random effects while modeling physical systems. An interesting method of that is to replace parameters in the deterministic model by random processes. In various cases, deterministic models frequently fluctuate due to noise, which is random or at least appears to be so. So, it is natural for us to move from deterministic problems to stochastic ones. In this framework, the stochastic differential equations (SDEs) can be used to characterize a response of such a model (see Refs. [1,29]). SDEs naturally refer to the time dynamics of the evolution of a state vector, based on the (approximate) physics of the real system, together with a driving noise process. The noise process can be assumed in several ways. It often symbolizes processes not included in the model, but present in the real system. Random differential and evolution systems play a crucial role in characterizing many social, physical, biological, medical and engineering problems [38,40]. SDEs are essential from the viewpoint of applications since they incorporate randomness into the mathematical description of phenomena thereby describing it more exactly. The qualitative properties of SDEs such as existence, controllability and stability for first-order stochastic differential equations have been investigated in several papers (see, for example Refs. [10,22,41]).

On the other hand, in many cases, it is advantageous to treat the second-order SDEs directly rather than to convert them to first-order systems. The second-order SDEs are the precise model in continuous time to account for integrated processes that can be made stationary. For example, it is beneficial for engineers to model mechanical vibrations or charge on a capacitor or condenser subjected to white noise excitation by means of a second-order SDEs. The deterministic type of second-order systems has been investigated (see [2,5,17,35] and references therein) while the stochastic type has been in growing state. Recently, much attention has been paid to second-order stochastic equations, and we cite the works [6,7,28,32] and the references therein.

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A lot of dynamical systems, which are classified with its characteristic corresponding to both continuous and discrete processes. These systems at certain instants of time are subjected to rapid changes symbolized by instantaneous jumps. Mathematical models for such systems are impulsive differential equations. It has proved to be valuable tools in the modeling of many phenomena in various fields of engineering and science. With regard to this issue; see the monographs of Lakshmikanthan et al. [24], Samoilenko and Perestyuk [36] and the references therein. Associated with this development, the area of impulsive differential equations significantly motivates a deeper theoretical study. Therefore, it is advantageous to study the theory of impulsive differential equations as a well-deserved discipline, due to its increase applications of many fields in the future. However, as well as impulsive effects, stochastic effects also exist in real systems. Several dynamical systems have variable structures subject to stochastic abrupt changes, which may result from abrupt phenomena such as sudden environment changes, changes in the interconnections of subsystems, stochastic failures and repairs of the components, etc. [25].

Differential delay equations or functional differential equations have been utilized in modeling scientific phenomena for many years. However, complicated circumstances in which the delay depends on the unknown functions have been proposed in modeling in recent years. These equations are often called equations with state-dependent delay (SDD). Functional differential equations with SDD perform frequently in applications as model of equations and thus have been discussed widely in the past years (see [8,16] and the references therein). Recently, existence and controllability results are derived for functional differential equations with SDD in [9,19,20,34]. On the other hand, controllability, as a central notions of mathematical control theory, plays an significant role in both deterministic and stochastic control systems. In general, controllability means that every state of a dynamical control system corresponding to a process can be affected or controlled in respective time by using some control signals. The existence and controllability result for nonlinear systems has been studied by many authors [2–4,27,31]. However, in several cases, some kind of randomness can appear in the problem. For this reason, the system should be modeled by a stochastic form. Motivated by these facts, our main purpose in this paper is to study the second-order impulsive stochastic evolution equations with SDD.

Precisely, consider the following abstract Cauchy problem

$$dx'(t) = [A(t)x(t)]dt + f(t, x_{\rho(t,x_t)})dw(t), \quad t \in J = [0, b], \ t \neq t_i, \ i = 1, 2, \ldots, n, \tag{1}$$

$$x_0 = \varphi \in \mathcal{B}, \quad x'(0) = \xi, \tag{2}$$

$$\triangle x|_{t=t_i} = I_i^1(x_{t_i}), \quad i = 1, 2, \ldots, n, \tag{3}$$

$$\triangle x'|_{t=t_i} = I_i^2(x_{t_i}), \quad i = 1, 2, \ldots, n. \tag{4}$$

Here, the stochastic process $x(\cdot)$ takes the values in a real separable Hilbert space H with inner product $(\cdot,\cdot)$ and the norm $\|\cdot\|$, and $A(t) : D(A(t)) \subseteq H \to H$ is a closed densely defined operator. $\{w(t)\}_{t\geqslant 0}$ is a given K-valued Brownian motion or Wiener process having a finite trace nuclear covariance operator $Q \geqslant 0$ defined on a complete probability space (Ω, F, P) equipped with a normal filtration $\{F_t\}_{t\geqslant 0}$ generated by w and K is another separable Hilbert space with inner product $(\cdot,\cdot)_K$ and the norm $\|\cdot\|_K$. Moreover, $L(K,H)$ will denote the space of all bounded linear operators from K into H endowed with the same norm $\|\cdot\|$. For $t \in J, x_t$ represents the function $x_t : (-\infty, 0] \to H$ defined by $x_t(\theta) = x(t+\theta), \ -\infty < \theta \leqslant 0$ which belongs to some abstract phase space $\mathcal{B}$ defined axiomatically. $f : J \times \mathcal{B} \to L_Q(K,H), \ \rho : J \times \mathcal{B} \to (-\infty, b]$ are appropriate functions and will be specified later. The impulsive moments t_i are given such that $0 < t_1 < \ldots < t_n < b, I_i^1(\cdot) : \mathcal{B} \to H$, $I_i^2(\cdot) : \mathcal{B} \to H, \ i = 1, 2, \ldots, n$. The symbol $\triangle\phi(t)$ represents the impulsive perturbation of $\phi(\cdot)$ at time t, defined by $\triangle\phi(t) = \phi(t^+) - \phi(t^-)$, where $\phi(t^+)$ and $\phi(t^-)$ are the right and the left limits of ϕ at t, respectively.

The literature related to existence and controllability of second order systems with impulses remains limited. Zhang et al. [42] established the sufficient conditions for the controllability of second-order semilinear impulsive stochastic neutral functional evolution equations by using the Sadovskii's fixed point theorem. In [2], the authors studied the controllability of damped second-order neutral functional differential systems with impulses in Banach spaces. Ren and Sun [30] discussed the second-order neutral stochastic evolution equations with infinite delay under Caratheodory conditions. Recently, existence results for impulsive neutral second-order stochastic evolution equations with nonlocal conditions in a real separable Hilbert space was considered in [11] with the aid of Sadovskii's fixed point theorem. The study of second-order evolution system with impulsive effect is not enormous and has to be paid some attention. The impulsive systems are fundamentally nonlinear and possess a number of specific effects caused by the occurrence of impulsive actions. These types of systems involve a wide area of applications in physics and mathematics. Also, the controller design procedure with SDD in practical engineering applications will be more important to the system stability and performance. However, it should be emphasized, to the best of our knowledge, the existence and controllability results for impulsive second order stochastic evolution systems with state-dependent delay in Hilbert space has not been investigated yet. Based on fixed point techniques, the proposed work in this paper on the second order stochastic evolution systems with impulsive effects and state-dependent delay is new in the literature. This fact is the principal goal of this work.

The structure of this article is as follows. In Section 2, some essential facts are recalled. Section 3 is devoted to the existence of mild solutions to problem (1)–(4). The controllability result is presented in Section 4. In Section 5, an example is provided to illustrate our results. We end this article with conclusion in Section 6.