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Impulsive control of sticking motion in van der Pol one-sided constraint system

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ABSTRACT

In this paper, the impulsive control method is used to dominate sticking motion in Van der Pol one-sided constraint system. We find that the method can restrain sticking motion effectively. And the method can also induce sticking motion for special parameters. In addition, numerical results show that the method is stable even for high levels of multiplicative noise or additive noise.

1. Introduction

There are many impact and dry friction nonsmooth factors in real world. The system is called nonsmooth system if it contains nonsmooth factors. Compared with smooth system, it shows strong nonlinearity and singularity. The bifurcations and chaos of nonsmooth systems are different from the form of smooth systems. Since these nonsmooth factors exist in this type of system, the theories of smooth systems cannot apply to nonsmooth systems. Some achievements have been gained [1–3], and the study of this field have been attracted many scholars [4–12].

Because of nonsmooth factors, the systems present complex dynamic behaviors that do not exist in the smooth systems. For example, sliding bifurcations [13–16], chatter and sticking motions are found in vibro-impact systems. Chris Budd and Felix Dux researched sticking motion of a single degree of freedom system in [17]. In Ref. [18] Toulemonde and Gontier discussed the periodic sticking motions in both single and multiple degree of freedom systems. The behavior of chatter, sticking and chaotic impacting motion in a two degree of freedom system has been studied by Wagg and Bishop in [19]. Wagg described rising phenomena and periodic sticking motion in [20,21], respectively. Sticking motions of a two degree of freedom system with multiple motion limiting constrains were considered in [22]. And Wagg also researched sticking motions of multiple degree of freedom vibro-impact system in [23]. Feng et al. discussed the chattering bifurcations in a Duffing unilateral vibro-impact system in [24].

Vibrating hammers and impact dampers are based on the impact for moving bodies. As we know, sticking motion can emerge in a vibro-impact system. So, it is worth to study the control of sticking motion. In the present paper, the impulsive control method is used to dominate the sticking motion of Van der Pol one-sided constraint system. Numerical results show the method can restrain sticking motion and induce sticking motion for special parameters. Furthermore, we consider the stability of the method.

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The paper is organized as follows. Section 2 presents the mathematical model of Van der Pol one-sided constraint system. The impulsive control method is introduced in Section 3. And the method is validated to be an effective way to restrict sticking motion. Moreover, we find that this method can induce sticking motion. Our conclusions are given in Section 4.

2. Theory model

In the paper, we mainly consider Van der Pol one-sided constraint system whose motion is governed by the following differential equation

$$\ddot{x} + (ax^2 - b)\dot{x} + x = F\cos(\omega t), \quad x < d, \tag{1a}$$

$$\dot{x}_+ = -r\dot{x}_-, \quad x = d, \tag{1b}$$

where a and b are positive parameters; F and ω are respectively the amplitude and frequency of harmonic excitation. When x meets constraint condition, the velocity of the system changes by Eq. (1b), where the subscripts "−" and "+" denote the velocity just before and after impact. Here r is the restitution coefficient, d is the non-dimensional distance.

The system parameters are chosen as $a = 0.5$, $b = 0.5$, $d = 2$, $r = 0.8$ and $F = 10$. The bifurcation diagram is shown in Fig. 1. As can be seen from the figure, the system undergoes incomplete chattering, complete chattering and sticking motion when ω decrease from 0.5 to 0.31. The process of sticking motion is the impact velocity reduces to zero when the sign of its acceleration does not change. It seems as if the body sticks the obstacle.

3. The control of sticking motion and its stability

3.1. Incomplete chatter and sticking motion

The phase trajectory diagram is shown in Fig. 2 on the condition that ω is chosen as 0.36. As the figure suggests, the orbit is periodic, and there are 5 collisions in one period. That is called incomplete chatter. When ω is selected as 0.31, the time series is displayed in Fig. 3. As shown in the figure, the sticking phenomena appear in the process of motion. In this section chatter and sticking motion are introduced by above two figures. In the following section, we primarily investigate the results of the control of sticking motion.

3.2. The control of sticking motion

As we know, the sticking phenomenon exists in the process of motion when $\omega = 0.31$. Under this condition, the impulsive control method is used to restrain sticking motion. The control function is chosen as $f = (1 - \lambda)y$, $y = \dot{x}_+$. Then

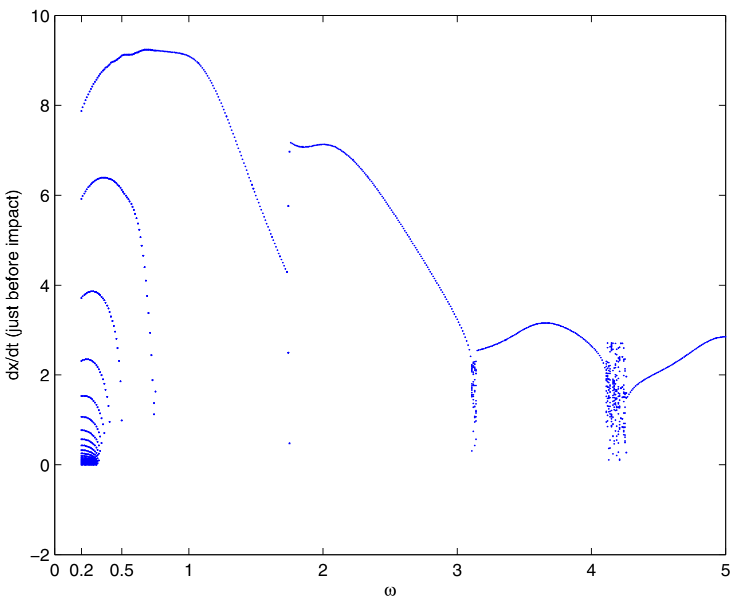

Fig. 1. Bifurcation diagram.