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Permanence and almost periodic solution of a discrete impulsive Richards growth equation with variable delays and feedback control

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ABSTRACT

In the old years, the researchers usually considered the permanence and almost periodic dynamics of the discrete time models without impulsive perturbations in biological populations. In this paper, we study a discrete impulsive Richards growth equation with variable delays and feedback control. By using some analysis technics and constructing a suitable Lyapunov functional, we investigate the permanence and global attractivity of the model. Based on the results of permanence and global attractivity, some sufficient conditions for the existence of a unique globally attractive positive almost periodic solution to the model are established, by the relation between the solutions of impulsive system and the corresponding non-impulsive difference system, and the almost periodic functional hull theory of difference system. To some extent, our main results complement and generalize some scientific work in recent years. Finally, some examples and numerical simulations are given to illustrate the effectiveness of our main results.

1. Introduction

For any bounded sequence $\{f(n)\}$ defined on $\mathbb{Z}$, $f^u \stackrel{\text{def}}{=} \sup_{n\in\mathbb{Z}}\{f(n)\}$, $f^l \stackrel{\text{def}}{=} \inf_{n\in\mathbb{Z}}\{f(n)\}$, $\bar{f} \stackrel{\text{def}}{=} \sup_{n\in\mathbb{Z}}\lfloor f(n)\rfloor$, $\underline{f} \stackrel{\text{def}}{=} \inf_{n\in\mathbb{Z}}\lfloor f(n)\rfloor$. $[a,b]_{\mathbb{Z}} \stackrel{\text{def}}{=} [a,b]\cap\mathbb{Z}$, $\forall a,b\in\mathbb{R}$. $\lfloor a\rfloor$ denotes the algebraically largest integer, which does not exceed a, $\forall a\in\mathbb{R}$.

In some cases, the traditional logistic model exhibits artificially complex dynamics. Therefore, it would be reasonable to get away from the specific logistic form in studying population dynamics and use more general classes of growth models. To drop an unnatural symmetry of the logistic curve, Berezansky and Idels [1] considered the following modified logistic form by Pella and Tomlinson [2–4] or the Richards' growth equation with delay:

$$x'(t) = x(t)\left\{r(t)\left[1-\left(\frac{x(t-\rho)}{K(t)}\right)^{\theta}\right]\right\}. \tag{1.1}$$

According to [4], $0<\theta<1$ is used for invertebrate populations (such as insects, worms, starfish, sponges, squid, plankton, crustaceans and mollusks), $\theta \geqslant 1$ is used for the vertebrate populations (such as amphibians, birds, fish, mammals and reptiles). In [1], the authors obtained oscillation and local stability results for Eq. (1.1).

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Discrete time model governed by difference equations are more appropriate than the continuous ones when the populations have non-overlapping generations. Discrete time models can also provide efficient computational models of continuous models for numerical simulations. In recent years, there are many papers concerning with the discrete time models in biological populations. In particular, the permanence and almost periodic dynamics of the discrete time models in biological populations have been extensively studied, see [5–15] and the references cited therein. For example, Li and Chen [13] considered the following discrete almost periodic logistic equation:

$$x(n+1) = x(n)\exp\left\{r(n)\left[1-\frac{x(n)}{K(n)}\right]\right\}. \tag{1.2}$$

Under the assumptions of almost periodicity of the coefficients of Eq. (1.2), the authors studied the existence, uniqueness and global attractivity of almost periodic solution of Eq. (1.2) by means of asymptotically almost periodic theory.

Population models with delays have attracted much attention in recent years. The application of delay equations to biological phenomena is associated with studies of dynamic phenomena like oscillations, bifurcations and chaotic behavior. Time delays represent an additional level of complexity that can be incorporated in a more detailed analysis of a particular system. Besides, ecosystems in the real world are often distributed by unpredictable forces which can result in changes in biological parameters such as survival rates, so it is necessary to study models with control variables which are so-called disturbance functions [7,9,11,14]. In [11], Li and Zhang considered the following discrete logistic equation with constant delays and feedback control:

$$\begin{cases} x(n+1) = x(n)\exp\left\{r(n)\left[1-\frac{x(n)}{K(n)}-\sum_{s_0=1}^{m_0} a_{s_0}(n)x(n-s_0)\right]-\sum_{s_1=0}^{m_1} b_{s_1}(n)u(n-s_1)\right\},\\ u(n+1) = (1-\alpha(n))u(n)+\sum_{s_2=0}^{m_2}\beta_{s_2}(n)x(n-s_2), \end{cases} \tag{1.3}$$

where $x(n)$ is the density of the species at time n and $u(n)$ is the control variable at time n. By applying the theory of difference inequality and constructing a suitable Lyapunov functional, the authors obtained some sufficient conditions which guarantee the permanence and existence of a unique globally attractive positive almost periodic sequence solution of system (1.3). For more works on almost periodic solution of the discrete systems with constant delays and feedback control, one could refer to [8–12] and the references cited therein.

The ecological system is often deeply perturbed by human exploitation activities such as planting and harvesting and so on, which makes it unsuitable to be considered continually. To obtain a more accurate description of such systems, we need to consider the impulsive differential equations. In recent years, the permanence and almost periodic dynamics of the impulsive biological models governed by functional differential equations have been intensively investigated (see [16–22] for more detail). However, to the best of authors' knowledge, there are few papers referring to the discrete time models with impulsive perturbations in biological populations. On the other hand, in real world, the delays in differential equations of biological phenomena are usually time-varying. Thus, it is very meaningful to study the discrete time models with impulsive perturbations and time-varying delays in biological populations.

Stimulated by the above reasons, in this paper, we consider the following discrete impulsive Richards growth equation with variable delays and feedback control:

$$\begin{cases} x(n+1) = x(n)\exp\left\{r(n)\left[1-\left(\frac{x(n-\lfloor\rho(n)\rfloor)}{K(n)}\right)^{\theta}-b(n)u(n-\lfloor\mu(n)\rfloor)\right]\right\}, \quad n\neq n_k,\\ x(n_k+1) = x(n_k^+)\exp\{r(n_k)F_{xu}(n_k)\}, \quad x(n_k^+) = \eta_k x(n_k),\\ u(n+1) = (1-\alpha(n))u(n)+\beta(n)x(n-\lfloor\nu(n)\rfloor), \end{cases} \tag{1.4}$$

where

$$F_{xu}(n_k) = \begin{cases} 1-\left(\frac{x(n_k-\lfloor\rho(n_k)\rfloor)}{K(n_k)}\right)^{\theta}-b(n_k)u(n_k-\lfloor\mu(n_k)\rfloor), \quad \lfloor\rho(n_k)\rfloor\neq 0,\\ 1-\left(\frac{x(n_k^+)}{K(n_k)}\right)^{\theta}-b(n_k)u(n_k-\lfloor\mu(n_k)\rfloor), \quad \lfloor\rho(n_k)\rfloor = 0, \quad k\in\mathbb{Z}. \end{cases} \tag{1.5}$$

Here θ is a positive constant, $\{r(n)\}$, $\{b(n)\}$, $\{\alpha(n)\}$, $\{\beta(n)\}$, $\{K(n)\}$, $\{\rho(n)\}$, $\{\mu(n)\}$ and $\{\nu(n)\}$ are bounded, nonnegative almost periodic sequences, $x(n_k^+)$ represents the impulsive perturbation at time n_k and $n_k < n_{k+1}$, $\lim_{k\to+\infty} n_k = +\infty$, $\{\eta_k\}$ is a nonnegative sequence.

Let $\delta \overset{\text{def}}{=} \max\{\bar{\rho},\bar{\mu},\bar{\nu}\}$. We consider system (1.4) together with the following initial condition

$$x(s) = \varphi(s), \quad u(s) = \phi(s), \quad \forall s\in[-\delta,0]_{\mathbb{Z}}, \quad \varphi,\ \phi\in C\big([-\delta,0]_{\mathbb{Z}},(0,+\infty)\big). \tag{1.6}$$

One can easily show that the solutions of system (1.4) with initial condition (1.6) are defined and remain positive for $n\in\mathbb{Z}^+ \overset{\text{def}}{=} [0,+\infty)_{\mathbb{Z}}$.

The sequence $z(n) = (x(n),u(n))^T : [-\delta,+\infty)_{\mathbb{Z}}\to\mathbb{R}^2$ is called a solution of system (1.4) with the initial condition (1.6) if it satisfies: