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Solutions of system of equilibrium and variational inequality problems on fixed points of infinite family of nonexpansive mappings ☆

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ABSTRACT

In this paper, we introduce a hybrid viscosity approximation scheme for finding a common element of the solution set for a system of equilibrium problems, the solution set for a system of variational inequality problems and the common fixed point set for an infinite family of nonexpansive mappings in Hilbert spaces. Then we prove the strong convergence of the proposed iterative scheme. Our results improve and extend the results announced by Ceng and Yao (2008) [4].

1. Introduction

Let C be a nonempty closed convex subset of a real Hilbert space H and let $T : C \to H$ be a nonlinear mapping, we denote by $Fix(T)$ the set of fixed point of T. Recall the following definitions:

(1) T is called α-contractive if there exists a constant $\alpha \in (0,1)$ such that

$$\|Tx - Ty\| \leqslant \alpha\|x - y\|^2, \quad \forall x, y \in C.$$

(2) T is called nonexpansive if

$$\|Tx - Ty\| \leqslant \|x - y\|, \qquad \forall x, y \in C.$$

(3) T is called λ-strictly pseudo-contractive of Browder and Petryshyn type [3] if there exists a constant $\lambda \in (0,1)$ such that

$$\|Tx - Ty\|^2 \leqslant \|x - y\|^2 + \lambda\|(I - T)x - (I - T)y\|^2, \quad \forall x, y \in C. \tag{1}$$

It is well-known that the last inequality is equivalent to

$$\langle Tx - Ty, x - y\rangle \leqslant \|x - y\|^2 - \frac{1 - \lambda}{2}\|(I - T)x - (I - T)y\|^2, \quad \forall x, y \in C.$$

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(4) T is called δ-inverse strongly monotone if there exists a positive real number $\delta > 0$ such that

$$\langle Tx - Ty, x - y\rangle \geqslant \delta\|Tx - Ty\|^2, \quad \forall x, y \in C.$$

It is obvious that any δ-inverse strongly monotone mapping is monotone and Lipschitzian.

The classical variational inequality problem is to find $x \in C$ such that

$$\langle Ax, y - x\rangle \geqslant 0, \quad \forall y \in C. \tag{2}$$

The set of solutions of (2) is denoted by $VI(C,A)$, that is,

$$VI(C,A) = \{x \in C : \langle Ax, y - x\rangle \geqslant 0, \quad \forall y \in C\}.$$

The variational inequality has been extensively studied in the literature. See e.g. [9–12] and the references therein.

Let F be a bi-function of $C \times C$ into $\mathbb{R}$, where $\mathbb{R}$ is the set of real numbers. The equilibrium problem for $F : C \times C \to \mathbb{R}$ is to determine its equilibrium points, i.e the set

$$EP(F) = \{x \in C : F(x, y) \geqslant 0, \quad \forall y \in C\}. \tag{3}$$

Let $\mathcal{J} = \{F_i\}_{i\in I}$ be a family of bi-functions from $C \times C$ into $\mathbb{R}$. The system of equilibrium problems for $\mathcal{J} = \{F_i\}_{i\in I}$ is to determine common equilibrium points for $\mathcal{J} = \{F_i\}_{i\in I}$, i.e the set

$$EP(\mathcal{J}) = \{x \in C : F_i(x, y) \geqslant 0, \quad \forall y \in C, \ \forall i \in I\}. \tag{4}$$

Numerous problems in physics, optimization, and economics reduce into finding some element of $EP(F)$. Some method have been proposed to solve the equilibrium problem; see, for instance [2,6,7,14,25]. The formulation (4), extend this formalism to systems of such problems, covering in particular various forms of feasibility problems [1,5].

Given any $r > 0$ the operator $J_r^F : H \to C$ defined by

$$J_r^F(x) = \left\{z \in C : F(z, y) + \frac{1}{r}\langle y - z, z - x\rangle \geqslant 0, \quad \forall y \in C\right\}$$

is called the resolvent of F, see [6]. It is shown in [6] that under suitable hypotheses on F (to be stated precisely in Section 2), $J_r^F : H \to C$ is single-valued and firmly non-expansive and satisfies

$$Fix(J_r^F) = EP(F), \quad \forall r > 0.$$

Using this result, in 2007, Takahashi and Takahashi [25] introduced an iterative scheme by viscosity approximation method for finding a common element of the set of solutions of an equilibrium problem and the set of fixed points of a non-expansive mapping in a Hilbert space. Let $T : C \to H$ be a non-expansive mapping. Starting with an arbitrary initial point $x_1 \in H$, they defined the sequence $\{x_n\}$ recursively by

$$x_{n+1} = \alpha_n f(x_n) + (1 - \alpha_n)TJ_{r_n}^F x_n,$$

where $\{\alpha_n\} \subset [0,1]$ and $\{r_n\} \subset (0,\infty)$ satisfy

$$\lim_{n\to\infty}\alpha_n = 0, \quad \sum_{n=1}^{\infty}\alpha_n = \infty, \quad \sum_{n=1}^{\infty}|\alpha_{n+1} - \alpha_n| < \infty,$$
$$\liminf_{n\to\infty} r_n > 0 \quad \text{and} \quad \sum_{n=1}^{\infty}|r_{n+1} - r_n| < \infty.$$

They proved that the sequences $\{x_n\}$ and $\{J_{r_n}^F x_n\}$ converge strongly to $z \in Fix(T) \cap EP(F)$, where $z = P_{Fix(T)\cap EP(F)}f(z)$ and $P_{Fix(T)\cap EP(F)}$ is the metric projection of H onto $Fix(T) \cap EP(F)$.

On the other hand, in 2008, Ceng and Yao [4] introduced and studied an implicit iteration process with perturbed mapping for finding a common fixed point of infinitely many non-expansive mappings in H. In addition, Yao et al. [27] introduced and considered an iterative scheme for finding a common element of the set of solutions of the equilibrium problem (3) and the set of common fixed points of infinitely many non-expansive mappings in H. Let $\{T_i\}_{i=1}^{\infty}$ be a sequence of non-expansive mappings of C into itself and let $\{\lambda_i\}_{i=1}^{\infty}$ be a sequence of nonnegative real numbers in $[0,1]$. For each $n \geqslant 1$, define a mapping W_n of C into itself as follows: