Applied Mathematics and Computation 248 (2014) 503–510

Contents lists available at ScienceDirect

Applied Mathematics and Computation

journal homepage: www.elsevier.com/locate/amc

Due date assignment and single machine scheduling with deteriorating jobs to minimize the weighted number of tardy jobs

Chuanli Zhao [a], Chou-Jung Hsu [b], Shuenn-Ren Cheng [c], Yunqiang Yin [d], Chin-Chia Wu [e,*]

[a] School of Mathematics and Systems Science, Shenyang Normal University Shenyang, Liaoning 110034, People's Republic of China
[b] Department of Industrial Engineering and Management, Nan Kai University of Technology, Nantou, Taiwan
[c] Graduate Institute of Business Administration, Cheng Shiu University, Kaohsiung County, Taiwan
[d] Faculty of Science, Kunming University of Science and Technology, Kunming 650093, People's Republic of China
[e] Department of Statistics, Feng Chia University, Taichung, Taiwan

ARTICLE INFO

Keywords:
Scheduling
Single-machine
Deteriorating jobs
Due date assignment

ABSTRACT

In this paper, we explore a single-machine scheduling problem in which the processing time of a job is a linear increasing function of its starting time. The objective is to determine the optimal due date and schedule simultaneously to minimize a cost function that includes the weighted number of tardy jobs and the due date assignment cost. We show that the problem is NP-hard in the ordinary sense. In addition, we propose two dynamic programming algorithms and a fully polynomial-time approximation scheme for the problem.

1. Introduction

Scheduling with deteriorating jobs was introduced by [3]. Since then, machine scheduling with deteriorating jobs has attracted increasing attention and many remarkable findings have been reported in this area [1,16,2,9,22,27–29]. Meeting due dates has always been one of the most important objectives in Just-In-Time systems. The study of models where the due date is a decision variable is motivated by the common real-life situation in which the due date is determined during sales negotiations with the customer. A number of recent studies viewed due date assignment as part of the scheduling process, and showed how the ability to control due dates can be a major factor in improving system performance (see [24]). For more applications of system performance with different settings, the reader may refer to [31,19].

Cheng et al. [5] studied a single-machine scheduling problem with linear deteriorating jobs. In their model, jobs have a common job-independent deterioration rate. The objective is to minimize the sum of the due-date, earliness and tardiness penalties. They showed that the optimal solution can be found in $O(n\log n)$ time. Kuo and Yang [17] examined a similar problem to that one considered by [5] and provided a simpler algorithm for the problem. Cheng et al. [6] considered the problem of assigning a common due date and scheduling deteriorating jobs on identical parallel machines. The objective is to minimize the sum of the due date, earliness and tardiness penalties. They showed that the problem is NP-hard, and presented a heuristic algorithm to find near-optimal solutions for the problem. Gawiejnowicz and Lin [10] presented a new model of time-dependent scheduling. The applied criteria of schedule optimality include the maximum completion time, the total

* Corresponding author.
E-mail address: cchwu@fcu.edu.tw (C.-C. Wu).

http://dx.doi.org/10.1016/j.amc.2014.09.095
0096-3003/

completion time, the total weighted completion time, the maximum lateness, and the number of tardy jobs. They delineated a sharp boundary between computationally easy and difficult problems. Li et al. [20] addressed the single- machine scheduling problem with deteriorating jobs and due date assignment. The objective was to minimize costs for the earliness, due date assignment and weighted number of tardy jobs. They presented polynomial-time algorithms that continued to solve the problem in the case of two popular due date assignment methods. Gawiejnowicz and Kononov [11] studied isomorphic problems that constitute a new class of mutually related scheduling problems. They first defined the class applying a one-to-one transformation of instances of a generic scheduling problem with fixed job processing times into instances of time-dependent scheduling problems with proportional-linear job processing time. Then they showed how to convert polynomial algorithms for scheduling problems with fixed job processing time into polynomial algorithms for time dependent scheduling problems with proportional-linear job processing times. Finally, they showed how approximation algorithms for isomorphic problems are related. For more details on scheduling with a common due date assignment, readers may refer to [21,4,25,13,12]. More recently, [30] addressed a double-layered optimization approach for the integrated due date assignment and scheduling problem. Elyasi and Salmasi [8] studied a due date assignment on a single machine with stochastic processing times.

2. Problem formulation and preliminaries

In this paper, we consider a single-machine scheduling problem with deteriorating jobs and a common due date assignment. The objective is to determine the optimal due date and simultaneously schedule jobs to minimize the sum of the weighted number of tardy jobs and due date assignment penalties. We assume that all jobs have to be assigned a common due date d, by which it is desirable to complete all jobs. If in some schedule a job completes later than common due date d, a penalty w_j is paid. The problem can be described as follows.

There is a set of n independent jobs $\mathcal{J} = \{J_1, J_2, \ldots, J_n\}$ which are simultaneously available at time zero. Job J_j is associated with a normal processing time a_j, and a deteriorating rate b_j. The actual processing time of job J_j is $p_j = a_j + b_jS_j$, where S_j is the starting time of J_j in a schedule, $a_j \geqslant 0, b_j \geqslant 0$, and $1 \leqslant j \leqslant n$. For job J_j, a weight w_j indicating its relative importance is given. All jobs have a common (but unknown) due date d. The objective is to determine the job sequence and common due date d to minimize a cost function that includes the weighted number of tardy jobs and the due date assignment cost which is defined by the following formulae:

$$\sum_{j=1}^{n} w_jU_j + \gamma d,$$

where C_j is the completion time of job J_j, U_j is the tardiness indicator variable for job J_j, i.e., $U_j = 1$ if $C_j > d$ and $U_j = 0$ if $C_j \leqslant d$, and γ represents the unit penalty for due date d. As same as in general scheduling model, we assume that all parameters of the problem are integers.

By using the general notation for scheduling problems which is proposed in book by Gawiejnowicz [9], the problem is denoted by

$$1|p_j = a_j + b_jS_j| \sum w_jU_j + \gamma d. \tag{1}$$

We first give some lemmas, which are useful to our subsequent analysis.

Lemma 1 [3]. *For the problem $1|p_j = a_j + b_jS_j|C_{\max}$, an optimal schedule can be obtained by sequencing the jobs in a non-decreasing order of a_j/b_j. If $\pi = [J_1, J_2, \ldots, J_n]$, the starting time of the first job is 0, then*

$$C_j = \sum_{j=1}^{n} a_j \prod_{i=j+1}^{n} (1 + b_i), \quad j = 1, \ldots, n.$$

Lemma 2. *For the problem $1|p_j = a_j + b_jS_j| \sum w_jU_j + \gamma d$, the optimal common due date for any job sequence is given by $d = C_{[l]} = \sum_{j=1}^{l} a_j \prod_{i=j+1}^{l} (1 + b_i)$ where $C_{[l]}$ is the completion time of a job in some position l ($l \in \{0, 1, \ldots, n\}$) in the sequence where $C_{[0]} = 0$.*

From Lemma 2, for any given job sequence of the problem $1|p_j = a_j + b_jS_j| \sum w_jU_j + \gamma d$, the optimal common due date is equal to the completion time of the last not tardy job.

As a result of Lemmas 1 and 2, we have following solution.

Lemma 3. *For the problem $1|p_j = a_j + b_jS_j| \sum w_jU_j + \gamma d$, there exists an optimal schedule such that not tardy jobs are scheduled in non-decreasing order of a_j/b_j.*