



Efficient bounds for the Monte Carlo–Neumann solution of stochastic systems



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ABSTRACT

The numerical solution of uncertainty propagation problems in mechanics can be computationally demanding. In this article, a well-known property of the Neumann series is explored in order to derive lower and upper bounds for expected value and autocorrelation of stochastic system responses. Uncertainties in system parameters are represented as parameterized stochastic processes. Monte Carlo simulation is employed to obtain samples of system response, from which lower and upper bounds of expected value and autocorrelation are computed. The proposed methodology is applied to two example problems, involving beam bending and axial thermo-elasticity. It is shown that accurate and efficient bounds can be obtained, for a proper choice of operator norm, with as few as one or two terms in the Neumann expansion. The Monte Carlo–Neumann bounding scheme proposed herein is shown to be an efficient alternative for the solution of uncertainty propagation problems in mechanics.

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1. Introduction

The last few decades have witnessed tremendous developments in the modeling of mechanical and structural systems, due to advances in computational mechanics. Numerical methods such as finite elements, finite difference, boundary elements, etc., have reached broad acceptability and wide coverage of applications. New developments address the solution of complex, non-linear problems. Multi-physics analyses allow investigation of new, unforeseen interaction effects between structures, soils, fluids, thermo-dynamic and electric effects. Significant developments have also been recently achieved in modeling uncertainty propagation through mechanical and other types of systems.

The Monte Carlo simulation method remains a popular, yet computationally expensive tool for analyzing uncertainty propagation through mechanical systems. The computational cost of Monte Carlo simulation can easily become prohibitive, for highly non-linear problems and complex geometries. Intrusive methods have recently been developed, such as the stochastic finite element method [1] or stochastic Galerkin Method [2–6]. Intrusive methods have the inconveniency of requiring full re-programming of conventional finite element software. Hence, non-intrusive Monte Carlo simulation methods remain popular in the solution of stochastic mechanics problems.

In linear stochastic mechanics problems, the numerical solution of a differential equation is replaced by the solution of a system of linear algebraic equations (stiffness matrix). In this context, when Monte Carlo simulation is employed, for each

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system realization, the stiffness matrix needs to be evaluated and inverted. Depending on the dimensions of the linear system, and the required number of samples, this can become computationally intensive. For a linear operator in finite dimensions, $\mathcal{A} : (\mathbb{R})^n \rightarrow (\mathbb{R})^n$, the inverse can be represented by the Neumann series, composed of operators $\mathcal{P} : (\mathbb{R})^n \rightarrow (\mathbb{R})^n$ related to $\mathcal{A}$. In finite dimensions, linear operators are matrices. The objective of using the Neumann series is to replace the matrix inversions by a truncated series expansion. However, depending on the number of terms in the Neumann series, the number of operations to be performed may become larger than required for the direct linear system solution, when some iterative method is used to compute the inverse. Therefore, in this paper, properties of the Neumann series are exploited in order to derive, formally, lower and upper bounds for each realization of the numerical approximation of the stochastic system response. To increase the efficiency of the proposed scheme, equivalence between norms of finite dimension operators are considered.

First use of the Neumann series to solve stochastic problems in mechanics goes back to the late eighties. Yamazaki [7] employed the Neumann series to obtain samples of the displacement response, for a plane elasticity problem with random Young's modulus. Araújo & Awruch [8] derived the response on non-linear static and dynamic problems, with random mechanical properties. Chakraborty & Dey [9] formulated the dynamic bending of curved beams, with uncertain stiffness parameters. Chakraborty & Dey [10,11] obtained expected value and variance of displacement responses considering uncertain geometries and mechanical properties. Lei & Qiu [12] employed the Neumann series to study uncertainty propagation in structures using movement equations. Chakraborty & Sarkar [13] estimated statistical moments of the transversal displacement response of curved beams resting on Winkler foundations. Chakraborty & Bhattacharyya [14] obtained response statistics for linear tri-dimensional elasticity problems, representing elastic properties as Gaussian processes of the continuous. Li et al. [15] presented a methodology to obtain solutions of linear systems, based on a discretization of the stochastic problem. Schevenels et al. [16] proposed a methodology based on Green functions, which was compared to the Neumann series, in a wave propagation problem on random Winkler foundation. In all references above, the Neumann series was employed in a conventional fashion, as an alternative to solve different stochastic problems. Convergence properties of the Neumann series were not explored in these articles.

The present article advances the state of the art by employing well-known convergence properties of the Neumann series to derive formal lower and upper bounds for the realizations of a stochastic system response. To increase the efficiency of the proposed scheme, equivalence between norms of finite dimension operators is considered. The developed scheme is illustrated by means of application to two example problems, involving beam bending and axial elasticity in a bar.

2. Linear stochastic uncertainty propagation problem

Several uncertainty propagation problems in mechanics are derived from well-known physical principles such as the first and second laws of thermo-dynamics, among other. In linear mechanics, equilibrium equations are generally derived from the minimization of certain functionals. One of the most popular is the "Principle of Minimum Potential Energy", for which the functional minimization yields the Euler–Lagrange equations and the boundary conditions. In this paper, we consider a linear elliptic boundary value problem, whose operator appears in beam and plate bending problems, as well as heat transfer in steady-state conditions. The stochastic problem to be addressed herein is defined in a probability space $(\Omega, \mathcal{F}, P)$, where " Ω " is the space of events, " $\mathcal{F}$ " is a σ -algebra of events and " P " is a probability measure. The linear uncertainty propagation problem consists in finding the system response (u), such that:

$$\left\{ \begin{array}{l} \text{Find } u \in L^2(\Omega, \mathcal{F}, P; (H^{2m}(D) \cap H^m(D))), \text{ such that :} \\ \sum_{|\alpha|, |\beta| \leq m} \partial_\alpha (\kappa_{\alpha\beta} \partial_\beta u)(x, \omega) = f(x, \omega), \forall (x, \omega) \in D \times (\Omega, \mathcal{F}, P), \text{ a.e.;} \\ \text{subject to specified boundary conditions} \end{array} \right. \quad (1)$$

where $f(\cdot)$ is the source term and $H^{2m}(D)$, $H^m(D)$ are Sobolev spaces. The stochastic system response is a consequence of the randomness in parameters $\{\kappa_{\alpha\beta}\}_{|\alpha|, |\beta| < m}$. In problems of mechanics, these parameters can be related to stiffness (e.g., beam bending) or thermal conductivity (heat transfer). In this paper, these parameters are represented as parameterized stochastic processes.

The following hypotheses are required to warrant existence and uniqueness of the solution:

$$\begin{aligned} H1 : \kappa_{\alpha\beta} &\in L^\infty(\Omega, \mathcal{F}, P; H^m(D)), \kappa_{\alpha\beta} > 0, \quad \text{a.e. in } D, \quad \forall \alpha, \beta \in \mathbb{N}^m; \\ H2 : f &\in L^2(\Omega, \mathcal{F}, P; L^2(D)). \end{aligned} \quad (2)$$

Hypothesis H1 ensures that parameters $\{\kappa_{\alpha\beta}\}_{|\alpha|, |\beta| < m}$ are differentiable, positive, and uniformly limited in probability [17]. Hypothesis H2 ensures that source terms have finite variance. From hypothesis H1 and H2, the Lax–Milgram lemma ensures existence and uniqueness of the solutions, for random samples of system parameters $\{\kappa_{\alpha\beta}\}_{|\alpha|, |\beta| < m}$. In this paper, a formal study about existence and uniqueness of the solutions will not be performed, in particular because we are so far dealing with a general uncertainty propagation problem in mechanics. Formal studies on existence and uniqueness of the solutions for beam bending problems are found in [2,3,6], for plate bending in [4] and for thermo-elasticity in [18].

Numerical solution of uncertainty propagation problems generally involves replacing the solution of a stochastic differential equation by the solution of a system of algebraic equations. The Galerkin method, one of the most popular techniques,

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