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A bio-inspired algorithm for identification of critical components in the transportation networks

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ABSTRACT

Critical components in a transportation or communication network are those which should be better protected or secured because their removal has a significant impact on the whole network. In such networks, they will be congested if they are being offered more traffic than it can process. In this paper, we employ principles of slime mould *Physarum polycephalum* foraging behaviour to identify the critical components in congested networks. When *Physarum* colonises a substrate, it develops a network of protoplasmic tube aimed at transporting nutrients and metabolites between distance parts of the cell. The protoplasmic network is continuously updating to minimize the transportation time, maximize the amount of cytoplasm pumped and minimize the overall length of the network. This optimization is achieved via a positive feedback between flux of cytoplasm and tube diameters. When a segment of a protoplasmic network is removed, the whole network reconfigures and thickness of tubes is updated till an equilibrium state is reached. The transient period from a disturbed state to an equilibrium state shows how critical the removed segment was. We develop a Physarum-inspired algorithm to identify critical links or nodes in a network by removing them from the network or calculating the transient period to new equilibrium state. The efficiency of the proposed method are demonstrated in numerical examples.

1. Introduction

The congestions induced by accidents is a common phenomenon in transport, telecommunication and power distribution networks [1–6]. To ease the congestion, we must identify critical components of the network responsible for the congestion [7–10]. By this way, we are capable of allocating the limited resources to improve or upgrade the corresponding components more efficiently. One of the approaches towards identifying critical components and easing congestion would be to adapt principles of reconfiguration observed in living systems.

In recent years, *Physarum* has been well-studied from a computational point of view. Nakagaki et al. found that *Physarum* is capable of finding the shortest path connecting two food sources in a given maze, as well as yielding reasonable solutions to network design in the Tokyo rail system in an efficient manner with low total cost, yet high transport efficiency and fault

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tolerance [11,12]. In what follows, Tero et al. [13] proposed a mathematical model to undercover the mechanism lying in the *Physarum* to find the shortest path. In 2012, Bonifaci et al. [14] proved that the mass of the *Physarum* will converge to the shortest path connecting two specified nodes in the network regardless of the initial mass distribution. To date, *Physarum* has been employed to deal with many graph theoretical problems, such as finding the shortest path [11,15–19], shortest path tree problem [20], connecting different arrays of food sources in an efficient manner [21–23], network design [12,24–27] and others [28–34].

In this paper, inspired by adaptive behaviour of *Physarum*, we develop an algorithm to identify critical components in congested networks. First of all, Physarum is employed to reach the equilibrium solution in the traffic network. Then we bring in a new parameter to measure the importance and rankings of the links and nodes from the eye of Physarum. Different from the approach in Ref. [35], the length associated with each link in the equilibrium stated is taken into consideration. Finally, we have considered the Braess paradox [36] and use our measure to identify the critical nodes/links with the varying of demand range. By comparing with the results in Ref. [35], we have demonstrated our approach is more effective and reasonable.

The remainder of this paper is organized as follows. In Section 2, we briefly introduce the basic theories, including the traffic network equilibrium model and the mathematical model of *Physarum polycephalum*. In Section 3, we present the proposed method in detail. In Section 4, we apply it into a simple transportation system to identify the critical components. Finally, we give a conclusion in Section 5.

2. Preliminaries

In this section, the basic theories, including the traffic network equilibrium model and the mathematical model of *Physarum polycephalum*, are briefly introduced.

2.1. Traffic network equilibrium model [37]

The traffic network equilibrium is a very fundamental problem in the transportation systems [38–42]. However, it plays a critical role when we design the transportation networks. It's defined as below: Consider a network G with the set of directed edges L with n_L elements. There are W pairs of origin/destination (O/D) routes with n_w elements. Let P_w be a set of acyclic paths joining O/D pair w, P be a set of acyclic paths for all O/D pairs, and n_P be a number of paths in this network. Links are denoted by a, b, etc; paths by p, q, etc, and O/D pairs by w_1, w_2, etc. d_w is a demand on amount of goods transported between O/D pair w $(w \in W)$.

Assume that a nonnegative flow on path p is represented by x_p, and the flow on link a is f_a. The path flows can be grouped into the vector $x \in R_+^{n_P}$ while the link flows can be grouped into the vector $f \in R_+^{n_L}$.

The following flow conservation must hold:

$$\sum_{p \in P_w} x_p = d_w, \quad \forall w \in W. \tag{1}$$

This means that the sum of the path flows associated with every path connecting each O/D pair must be equal to the demand for that O/D pair.

Similarly, the flow on a link must be equal to the sum of the flows on path containing that link. Therefore, the following must hold:

$$f_a = \sum_{p \in P} x_p \delta_{ap}, \quad \forall a \in L, \tag{2}$$

where $\delta_{ap} = 1$, it denotes that link a is contained in the path P. Otherwise, it will not be contained in path p.

The travel cost on a path p is expressed as C_p while the travel cost on a link a is c_a. The cost on a path is the sum of the costs of links in the path:

$$C_p = \sum_{a \in L} c_a \delta_{ap}, \quad \forall p \in P, \tag{3}$$

where $\delta_{ap} = 1$, if link a is in the path p, and $\delta_{ap} = 0$, otherwise.

We consider networks to be prone to congestion, i.e. the travel cost function on each link depends on the links flows:

$$c_a = c_a(f), \quad \forall a \in L. \tag{4}$$

We assume that the link cost functions are continuous and monotonically increasing. According to Eqs. (1)–(3), we have

$$C_p = C_p(x), \quad \forall p \in P. \tag{5}$$

The network equilibrium is defined as follows. A path flow $x^* \in K^1$, where $K^1 \equiv \{x | x \in R_+^{n_P}$ and (1) holds$\}$, is said to be a network equilibrium, if the following conditions hold for each O/D pair $w \in W$ and each path $p \in P_w$:

$$C_p(x^*) \begin{cases} = \lambda_w, & \text{if } x_p^* > 0, \\ \geqslant \lambda_w, & \text{if } x_p^* = 0. \end{cases} \tag{6}$$