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On some time non-homogeneous queueing systems with catastrophes

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ABSTRACT

Non-stationary queueing systems subject to catastrophes occurring with time varying intensity are considered. The effect of a catastrophe is to make the queue instantly empty. The transition probabilities, the related moments and the first visit time density to zero state are analyzed. Particular attention is dedicated to queueing systems in the presence of catastrophes with periodic intensity function. Various applications are provided, including the non-stationary birth–death process with immigration, the queueing systems $M(t)/M(t)/1$ and $M(t)/M(t)/\infty$.

1. Introduction

Continuous-time Markov chains are used as models in various fields such as queueing systems, neuroscience, mathematical biology and finance. Recently these processes have been studied assuming that they are subject to total catastrophes. Specifically, a catastrophe is an event that occurs at random times and produces the instantaneous clearing of the system. So that each catastrophe causes a jump of the process from current state to zero state. The catastrophes play an interesting role in various areas of science and technology including service systems and population dynamics. Specifically, in a queue a catastrophe can describe a failure implying the reset of the system; whereas in population dynamics a catastrophe can be caused by an epidemic or an extreme natural disaster (forest fire, flood,...).

The contributions in the continuous-time Markov chains in the presence of catastrophes are various. They concern the determination of distributions and of other quantities of interest for continuous-time Markov chains subject to catastrophes (see, for instance [1–7]). Further results are related to the analysis of the effect of jumps in Markov queueing systems, including the case when the number of initially present customers is random (see, for instance [8–11]). Moreover, in [12,13] quantities of interest are derived when catastrophes reduce the population size by a random amount which is either geometrically, binomially or uniformly distributed. In particular, in [13] several approaches for the transient analysis of the continuous-time Markov chains in the presence of catastrophes have been discussed.

Taking into account that many of the real phenomena are non-stationary, we consider a queueing system whose rates are time-dependent as well as they are dependent on the number of the customers present in the queue (see, for instance [14–19]). The evolution of the system is subject to the effect of disasters that occur at random times and that empty instantaneously the system reducing to zero the number of customers.

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In Section 2 a inhomogeneous continuous-time Markov chain subject to total catastrophes with time varying intensity is considered. The analysis of such a system is performed by studying the transition probabilities and the moments of the number of the customers in the system. The problem of the first visit time (FVT) to zero state is also analyzed, with particular attention to busy period of the service center, i.e the time interval during which at least one server is busy. In Section 3 we focus our attention to the case in which the catastrophe intensity is a periodic function of time. Some properties of asymptotic distribution and the FVT density are proved. Sections 4–6 are dedicated to some applications in the context of queueing systems. In particular, in Section 4 the effect of total catastrophes is analyzed for a time non-homogeneous birth–death process with immigration. In Sections 5 and 6 we study the $M(t)/M(t)/1$ queueing system and the multiserver queueing system $M(t)/M(t)/\infty$, both with time dependent catastrophes rates. Extensive numerical computations with MATHEMATICA have been performed to show the role played by the involved intensities of arrival, departure and catastrophe.

2. Continuous Markov chains subject to total catastrophes

Let $\{N(t),\, t \geqslant t_0\}$ be a non-homogeneous continuous Markov chain with jumps, representing a queueing system in the presence of catastrophes. We suppose that the catastrophes occur according to a time-non-homogeneous Poisson process. The state space for $N(t)$ is $\mathcal{S} = \{0, 1, 2, \ldots\}$. We assume that $N(t)$ is regulated from transitions that occur according to the following scheme:

- $n \to n + 1$ with rate $\alpha_n(t)$, for $n = 0, 1, \ldots$,
- $n \to n - 1$ with rate $\beta_n(t)$, for $n = 2, 3, \ldots$,
- $1 \to 0$ with rate $\beta_1(t) + \xi(t)$,
- $n \to 0$ with rate $\xi(t)$, for $n = 2, 3, \ldots$,

where $\alpha_n(t) > 0,\ \beta_n(t) > 0$ and $\xi(t) \geqslant 0$ are bounded and continuous functions of the time. As illustrated in Fig. 1, arrivals occur with rates $\alpha_n(t)$, departures with rates $\beta_n(t)$, and catastrophes with rate $\xi(t)$. For the process $N(t)$ it is interesting to study some statistical characteristics as the transition probabilities, the related moments and the distribution of the random variable FVT to zero state.

For $j, n \in \mathcal{S}$ and $t > t_0 \geqslant 0$ the transition probabilities $p_{j,n}(t|t_0) = \mathrm{P}\{N(t) = n\,|\,N(t_0) = j\}$ $(j, n \in \mathcal{S})$ satisfy the following system of forward equations:

$$\frac{d}{dt}p_{j,0}(t|t_0) = -[\alpha_0(t) + \xi(t)]\,p_{j,0}(t|t_0) + \beta_1(t)\,p_{j,1}(t|t_0) + \xi(t),$$

$$\frac{d}{dt}p_{j,n}(t|t_0) = -[\alpha_n(t) + \beta_n(t) + \xi(t)]\,p_{j,n}(t|t_0) + \alpha_{n-1}(t)\,p_{j,n-1}(t|t_0) + \beta_{n+1}(t)\,p_{j,n+1}(t|t_0) \quad (n = 1, 2, \ldots) \tag{1}$$

with initial condition $\lim_{t\downarrow t_0} p_{j,n}(t|t_0) = \delta_{j,n}$. We denote by $\{\widetilde{N}(t),\, t \geqslant 0\}$ the time non-homogeneous continuous-time Markov chain obtained from $N(t)$ by removing the possibility of catastrophes, i.e. when $\xi(t) \downarrow 0$ for $t \geqslant t_0$. The transition probabilities $\widetilde{p}_{j,n}(t|t_0) = \mathrm{P}\{\widetilde{N}(t) = n|\widetilde{N}(t_0) = j\}$ $(j, n \in \mathcal{S})$ then satisfy the system of forward equations obtained from (1) by taking $\xi(t) \downarrow 0$ for $t \geqslant t_0$, with initial conditions $\lim_{t\to t_0} \widetilde{p}_{j,n}(t|t_0) = \delta_{j,n}$.

Hereafter we shall restrict our attention to non-explosive processes $\widetilde{N}(t)$, i.e.

$$\sum_{n=0}^{+\infty} \widetilde{p}_{j,n}(t|t_0) = 1, \quad j \in \mathcal{S},\ t \geqslant t_0$$

and we shall assume that

$$\lim_{t\to+\infty} \int_{t_0}^{t} \xi(u)\, du = +\infty. \tag{2}$$

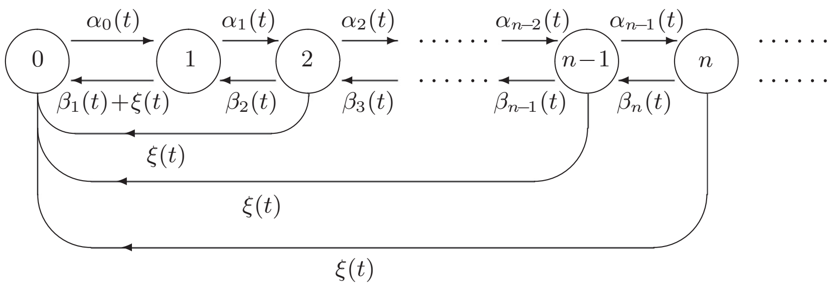

Fig. 1. The state diagram of the process $N(t)$.