Applied Mathematics and Computation 245 (2014) 243–254

Contents lists available at ScienceDirect

Applied Mathematics and Computation

journal homepage: www.elsevier.com/locate/amc

Stabilisation of mode-dependent singular Markovian jump systems with generally uncertain transition rates ☆

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ARTICLE INFO

Keywords:
Singular Markovian jump system
Generally uncertain transition rate
Stochastic stability
Stochastic admissibility

ABSTRACT

This paper is devoted to investigating the stability and stabilisation problems for continuous-time mode-dependent singular Markovian jump systems (SMJSs) with generally uncertain transition rates (GUTRs). First, we establish a sufficient condition in terms of a set of coupled linear matrix inequalities (LMIs) to ensure the systems to be regular, impulse-free and stochastically stable. Then, we design a mode-dependent state-feedback controller to guarantee the closed-loop systems stochastically admissible by applying the LMI technique. Finally, a numerical example is presented to illustrate the effectiveness and efficiency of the proposed method.

1. Introduction

Singular systems, also referred to as descriptor systems, generalized state-space systems, differential–algebraic systems or semi-state systems, appear in many systems, such as engineering systems (for example, power systems, electrical networks, aerospace engineering, chemical processes), social economic systems, network analysis, biological systems, and so on. Basic control theory for singular systems has been widely studied, such as stability and stabilization [1,3,23], optimal control [11], H_∞ control problem and filtering problem [26].

Markovian jump systems (MJSs), as a special class of stochastic hybrid systems, have been attracting increasing attention in the past few decades, see [8–10,13,16,31] and the references therein. Recently, the problem of stochastic stability and stochastic admissibility for singular Markovian jump systems (SMJSs) have attracted more and more discussions [2,14,15,19,21,22]. However, the transition rates (TRs) in the above mentioned literatures are assumed to be completely known. In practice, the TRs in some jumping processes are impossible to precisely estimate [4,5,7]. Karan et al. [7] considered the stochastic stability robustness for continuous-time and discrete-time Markovian jump linear systems (MJLSs) with upper bounded TRs. Zhang and Boukas [29] discussed stability and stabilization for the continuous-time MJSs with partly unknown TRs. Zhang and Lam [32] obtained necessary and sufficient conditions for analysis and synthesis of Markov jump linear systems with incomplete transition descriptions. Partly unknown TRs for MJSs were also involved in [12,17,18,20,24,25,28,31].

☆ The first author is supported by the National Natural Science Foundations of China (61272077), NCET-08-0755, National 863 Plan Project (2008 AA04Z401, 2009AA043404), the Natural Science Foundation of Shandong Province (No. Y2007G30), the Scientific and Technological Project of Shandong Province (No. 2007GG3WZ04016), the Natural Science Foundation of Guangxi Autonomous Region (No. 2012GXNSFBA053003), the Natural Scientific Research Innovation Foundation in Harbin Institute of Technology (HIT.NSRIF. 2001120).

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http://dx.doi.org/10.1016/j.amc.2014.06.064
0096-3003/

Wei et al. [20] developed a new H_∞ filtering design for continuous-time Markovian jump systems with time-varying delay and partially accessible mode information. The authors in [30] investigated mode-dependent H_∞ filtering for discrete-time Markovian jump linear systems with partly unknown transition probability. Unfortunately, the complete knowledge of every TR in the above mentioned models is either exactly known or completely unknown, which may be too restrictive in many practical situations. Guo and Wang [6] proposed another description for the uncertain TRs, which is called generally uncertain TRs (GUTRs). In this case, each transition rate can be completely unknown or only its estimate is known, which makes the GUTR model be applicable to more practical cases. Both bounded uncertain TR models and partly uncertain TR models are the special cases of GUTR models. To the best of our knowledge, the stabilisation problems for mode-dependent SMJSs with GUTRs has not been investigated, which is still important and challenging in many practical applications.

In this paper, we will investigate the problem of stochastic admissibility and stabilisation for SMJSs with generally uncertain TRs. In Section 2, the SMJS model with generally uncertain TRs is formulated and some definitions and lemmas are stated. In Section 3, we first establish a sufficient condition such that the unforced SMJS model with GUTRs is stochastically admissible, and then we design a linear state-feedback controller for the SMJS model with GUTRs such that the closed-loop system is stochastically admissible. In Section 4, a numerical example is provided to illustrate the feasibility and applicability of the developed results. Section 5 concludes the paper.

Notation. In this paper, $\Re^n$ and $\Re^{n\times m}$ denote n-dimensional Euclidean space and the set of all $n\times m$ real matrices respectively. The superscript T stands for matrix transposition. N^+ represents the set of positive integers. The notation $P>0$ ($P\geqslant 0$) means that the matrix P is a real symmetric and positive definite (semi-positive-definite) matrix. For notation $(\Omega,\mathbb{F},P)$, Ω represents the space, $\mathbb{F}$ is the σ-algebra of the sample space and P is the probability measure on $\mathbb{F}$. $\mathbb{E}\{\cdot\}$ stands for the mathematical expectation. Matrices, if their dimensions are not explicitly stated, are assumed to be compatible for algebraic operations.

2. Preliminaries and problem formulation

Consider the following continuous-time mode-dependent descriptor Markovian jump systems described by

$$E(r_t)\dot{x}(t)=A(r_t)x(t)+B(r_t)u(t), \tag{1}$$

where $x(t)\in\Re^n$ is the system state and $u(t)\in\Re^m$ is the input vector. The matrix $E(r_t)$ may be singular, without loss of generality, we can assume that, $\mathtt{rank}(E(r_t))=n_{E(r_t)}<n$. The mode jumping process $\{r_t,t\geqslant 0\}$ is a right-continuous Markov process taking values in a finite state-space $\mathbb{S}=\{1,2,\ldots,s\}$ with the following mode transition probabilities

$$Pr\{r_{t+\Delta}=j|r_t=i\}=\begin{cases}\pi_{ij}\Delta+o(\Delta), & i\neq j,\\ 1+\pi_{ij}\Delta+o(\Delta), & i=j,\end{cases} \tag{2}$$

where $\Delta>0, \lim_{\Delta\to 0}\frac{o(\Delta)}{\Delta}=0$, and $\pi_{ij}\geqslant 0, (i\neq j)$ is the transition rate from mode i at time t to mode j at time $t+\Delta$, and there is

$$\pi_{ii}=-\sum_{j=1,j\neq i}^{s}\pi_{ij}\leqslant 0.$$

The initial condition of the system state is $x(0)\triangleq x_0$. The system matrices $E_i\triangleq E(r_t=i),\ A_i\triangleq A(r_t=i)$ and $B_i\triangleq B(r_t=i)$, for $\forall i\in\mathbb{S}$, are known real constant matrices.

Remark 1. Singular systems are popular in modeling engineering systems (for example, power systems, electrical networks, aerospace engineering, chemical processes), social economic systems, network analysis, biological systems, and so on. The Markovian jump singular system model (1) is reasonable, because many physical systems may happen abrupt variations in their structure, due to random failures or repair of components, sudden environmental disturbances, changing subsystem interconnections, abrupt variations in the operating points of a nonlinear plant.

In this paper, the mode TR matrix $\Pi\triangleq(\pi_{ij})_{s\times s}$ are considered to be generally uncertain. For instance, the TR matrix of system (1) with s operation modes may be expressed as

$$\begin{bmatrix}\hat{\pi}_{11}+\Delta_{11} & ? & ? & \cdots & ?\\ ? & ? & \hat{\pi}_{23}+\Delta_{23} & \cdots & \hat{\pi}_{2s}+\Delta_{2s}\\ \vdots & \vdots & \vdots & \ddots & \vdots\\ ? & \hat{\pi}_{s2}+\Delta_{s2} & ? & \cdots & ?\end{bmatrix} \tag{3}$$

where $\hat{\pi}_{ij}$ and $\Delta_{ij}\in[-\delta_{ij},\delta_{ij}](\delta_{ij}\geqslant 0)$, represent the estimate value and estimate error of the uncertain transition rate π_{ij} respectively, and $\hat{\pi}_{ij},\delta_{ij}$ are known. "?" represents the complete unknown transition rate π_{ij}, which means its estimate value $\hat{\pi}_{ij}$ and estimate error bound δ_{ij} are unknown.