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How much information should we drop to become intelligent?

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ABSTRACT

Cognitive processing by intelligent systems involves the deletion of information in favor of higher level abstractions. This process can be addressed through the physics of computation but a formal model that explains this process has not been proposed yet. In this short paper, we propose a model that through physical constraints only generates optimal solution to the collapse of n objects into n sets. A numerical simulation of the model results in a logarithmic function of information loss and condensation that perfectly fits our knowledge of cognitive processes.

1. Introduction

Intelligence is usually conceived in simple additive terms. For instance, the more declarative-factual knowledge one possesses the more intelligent s(he) is considered to be.

However, a remarkable characteristic of intelligent systems is that they involve the erasure of information in favor of higher level abstractions. One who holds a large amount of information without being able to drop some of it in favor of abstractions is an *Idiot Savant*. This 'Savant Syndrome' is beautifully illustrated in Borges famous story "Funes the Memorious" [1] in which a person who has an incredible and unlimited memory cannot think beyond the myriad concrete details that populate his memory.

Cognition is mainly about producing categories and operating on categories [2] and categories are abstractions that involve the loss of information. When a child develops the category of Cats, for instance, he ignores many details of the particular cats that he has encountered in favor of the abstract category that has no tail or mustache. Despite the differences between particular cats they are grouped into a single category which is different from the Dogs category for instance.

The process of erasing information is favor of abstractions exists in a variety of living systems and for a good physical reason [3]. In the physics of computation, it has been argued by Bennett and Landauer [4] in one of their more philosophical papers that information should be equated with *differences* and that a process of computation, which is an irreversible process of producing an output from an input necessarily involves the loss of information/differences.

Given that any intelligent system necessarily involves the loss of information, one may ask how much information should we drop to become "intelligent" and more generally how can we model the loss of information in the formation of categories. This question has interesting consequences not only for our understanding of human and non-human cognition but for the design of intelligent systems specifically in the age of Big Data.

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Nomenclature

n	number of objects
x_i	objects $i = 1, \ldots, n$
E	energy function of the system
T	temperature of the system
k	Boltzmann's constant
Z	partition function
Γ	phase space
S	entropy
ϕ	probability density function (PDF)
$S(,)$	Stirling number

2. The theoretical approach

In this paper, we try to address the above challenge by providing a formal model of categories formation and information loss. Although intensive work has been conducted on data categorization, our paper follows a totally different line by studying the mathematical constraints imposed on set partition regardless of real world applications.

Our starting point is by considering abstraction as the *collapse of several differences into a single set*. More specifically, we approach the problem of abstraction through the partitions of a set where each partition is considered to be a category.

The lattice formed through this process includes an upper and lower bound that are clearly uninformative. For instance, let's imagine a universe that includes only 4 objects. A universe in which all objects are members of the same set is an undifferentiated universe with no informative value, the same as the universe in which each and every object stands "in-and-for-itself" to use an old philosophical phrase coined by Husserl. Hence, cognition and intelligence exist *in-between* these upper and lower bounds of the lattice.

The total number of partitions of an n objects set is calculated through the Bell Number. The Bell Number gives us the problem space that covers all potential partitions. However, an intelligent system cannot use this mathematical space as it grows exponentially in a way which is far beyond the limits of efficient cognitive processing computation. For example, for a set of five elements there are 52 possible partitions. It is unreasonable to assume that an intelligent system that processes n objects, automatically computes the Bell Number for all possible partitions. Therefore, while mathematics gives us the potential space of set partitions, at least as an analytical frame of reference, we must move to the physical realm in order to impose real-world constraints on the number of possible partitions and the way information is lost while collapsing differences into categories. The next section develops the physical–mathematical model of this process with the aim of better understanding information loss in abstraction as evident in the formation of categories.

3. The model, the simulations and the results

We start with the basic assumption that one should "pay" in energetic terms for a refinement partition of a given set. In other words, for collapsing objects into subsets we pay a price. Hence, for a system processing n objects and categorizing them into n categories/sets the question is how to optimally behave. Our model draws on statistical mechanics [11,12].

We start with a set of objects $x_1, x_2, \ldots, x_n$. For these sets we define the following probability density function:

$$\phi(x_1, x_2, \ldots, x_n) = e^{-E(x_1, x_2, \ldots, x_n)/kT}/Z, \tag{3.1}$$

where the partition function Z is given by:

$$Z = \int_\Gamma e^{-E(x_1, x_2, \ldots, x_n)/kT} dx_1 dx_2 \ldots dx_n. \tag{3.2}$$

We define the partition function through the integral because our phase space, Γ, is defined by the Bell Number that may be approximated by the integral. Our next step is to define the Gibbs Entropy of the system for each group of subsets i.e., level in Hasse diagram as follows:

$$S_i = -k \int_{\Gamma_i} \phi_i(x_{i_1}, x_{i_2}, \ldots, x_{i_m}) ln(\phi_i(x_{i_1}, x_{i_2}, \ldots, x_{i_m})) dx_{i_1} dx_{i_2} \ldots dx_{i_m}, \tag{3.3}$$

where Γ_i is the phase space defined by the Stirling number of the second kind. Let S_{In} and S_{Out} be the Entropy associated with the upper and the lower nodes of the Hasse diagram. Because of the symmetry between Γ_i and Γ_{i+1} the partition function for the i level in the Hasse diagram is twice the partition function of the $i+1$ level. Hence $\phi_i = \phi_{i+1}/2$ and