Applied Mathematics and Computation 245 (2014) 372–381

Contents lists available at ScienceDirect

Applied Mathematics and Computation

journal homepage: www.elsevier.com/locate/amc

Numerical solution of the coupled viscous Burgers equations by Chebyshev–Legendre Pseudo-Spectral method

Abdur Rashid [a,*], Muhammad Abbas [b,*], Ahmad Izani Md. Ismail [c], Ahmad Abd Majid [c]

[a] Department of Mathematics, Gomal University, 29100 Dera Ismail Khan, Pakistan
[b] Department of Mathematics, University of Sargodha, 40100 Sargodah, Pakistan
[c] School of Mathematical Sciences, Universiti Sains Malaysia, 11800 USM Penang, Malaysia

ARTICLE INFO

Keywords:
Coupled viscous Burgers equations
Chebyshev–Legendre Pseudo-Spectral method
Semi-discrete scheme and fully-discrete scheme

ABSTRACT

In this paper, we consider Chebyshev–Legendre Pseudo-Spectral (CLPS) method for solving coupled viscous Burgers (VB) equations. A leapfrog scheme is used in time direction, while CLPS method is used for space direction. Chebyshev–Gauss–Lobatto (CGL) nodes are used for practical computation. The error estimates of semi-discrete and fully-discrete of CLPS method for coupled VB equations are obtained by energy estimation method. The numerical results of the present method are compared with the exact solution for two test problems.

© 2014 Elsevier Inc. All rights reserved.

1. Introduction

The coupled VB equations was derived by Esipov [1] to study the model of sedimentation. The partial differential equations (PDEs) of coupled VB equations are as follow:

$$\frac{\partial \Psi}{\partial t} - \frac{\partial^2 \Psi}{\partial x^2} + \gamma \Psi \frac{\partial \Psi}{\partial x} + \alpha \frac{\partial}{\partial x}(\Psi \Phi) = 0, \quad x \in \Omega, \ t \in [0, T], \tag{1.1}$$

$$\frac{\partial \Phi}{\partial t} - \frac{\partial^2 \Phi}{\partial x^2} + \delta \Phi \frac{\partial \Phi}{\partial x} + \beta \frac{\partial}{\partial x}(\Psi \Phi) = 0, \quad x \in \Omega, \ t \in [0, T], \tag{1.2}$$

with initial conditions

$$\Psi(x, 0) = f(x), \quad \Phi(x, 0) = g(x), \quad x \in \Omega, \tag{1.3}$$

and the boundary conditions

$$\Psi(-L, t) = \Psi(L, t), \quad \Phi(-L, t) = \Phi(L, t), \quad t \in [0, T], \tag{1.4}$$

where $\Omega = [-L, L]$, γ and δ are real constants, α and β are arbitrary constants.

There has been continued interest in the solution of the VB equations. La Bryer et al. [2] developed a new method optimal spatiotemporal reduced order modelling to improve the order of accuracy for VB equations. Huilin and Chang Feng [3] solved the coupled VB equations by using new lattice Boltzmann model in 2014. They investigated the accuracy and stability of the new model in detail. Hossein used Laplace transform and homotopy perturbation methods to find the analytical approximation for the coupled VB equations [4].

* Corresponding authors. Mobile: +92 304 6282830 (M. Abbas).
E-mail addresses: prof.rashid@yahoo.com (A. Rashid), m.abbas@uos.edu.pk (M. Abbas).

http://dx.doi.org/10.1016/j.amc.2014.07.067
0096-3003/© 2014 Elsevier Inc. All rights reserved.

Mittal and Arora [5] proposed a numerical solution by using cubic spline collocation technique in 2011. A numerical technique is used to control the boundary conditions for VB equation by Ismail and Ibrahim [6] in 2009. A difference integral method was proposed by Jerez [7] for the numerical solution of VB equation in 2008. Horsin solved the VB equation by using Lagrangian controllability method in [8]. Nejib [9] developed a numerical method for boundary and distributed control of VB equation. A domain decomposition method was applied to two-dimensional VB equation by Louise Perkins and Rodrigue [10] in 1990. Rashid and Ismail [11] employed Fourier pseudospectral method for solving coupled VB equations.

Several numerical methods have been developed and implemented for solving nonlinear partial differential equations by many authors such as: Fakhar-Izadi and Dehghan [12] employed pseudospectral Legendre–Galerkin method for solving nonlinear integro-differential equation. Shamsi and Dehghan [13] and Salahi and Dehghan [14] solved three-dimensional parabolic equations and nonlinear Hamilton–Jacobi equations respectively by Legendre pseudospectral method. Shakeri and Dehghan [15] and Fakhar-Izadi and Dehghan [16] developed spectral method for solving nonlinear wave equations and Volterra integro-differential equations respectively. Dehghan used different numerical techniques like Spectral collocation method [17], Chebyshev pseudospectral multi-domain method [18], Mixed finite difference and Galerkin methods [19], time–space pseudospectral method [20] and Jacobi matrix method to different nonlinear partial differential equations and ordinary differential equations [21].

In our present study, we considered CLPS method for solving coupled VB equations. Error estimation of semi-discrete and fully-discrete are obtained by energy estimation method. We follow the work of Zhang and Xinmin [27], they found the error estimation of semi-discrete and fully discrete schemes for pseudospectral method for a class of system of LS wave interaction.

The outline of present study is as follows: In Section 2, some preliminaries are defined for solving viscous Burgers equations. The semi-discrete error estimates are obtained in Section 3, while error estimates of fully discrete scheme are given in Section 4. Numerical results are presented in Section 5. Finally conclusion is included in Section 6.

2. Preliminaries

The norm and inner product are defined by $\|\Psi\|^2 = (\Psi, \Psi)$ and $(\Psi, \Phi) = \int_\Omega \Psi(x)\Phi(x)dx$ respectively. The maximum norm is denoted by $\|\Psi\|_\infty$. The periodic Sobolev spaces is defined by

$$H_p^m = \{\Psi \in H^m(R) : \Psi(x - L) = \Psi(x + L)\},$$

where m is positive integer, the Sobolev norms are defined by

$$\|\Psi\|_m^2 = \sum_{j=0}^{m} \left\| \frac{\partial^j \Psi}{\partial x^j} \right\|^2.$$

S_N denotes the space of trigonometric polynomials of degree up to N

$$S_N = \text{span}\left\{ \frac{1}{\sqrt{L}} \exp\left(\frac{i\pi x_j}{L} \right) : j = 0, 1, \ldots, N \right\},$$

where $i = \sqrt{-1}$. P_N denotes the L^2 orthogonal projection operator of $H_p^m(\Omega)$ upon S_N. The interpolation operator at the Chebyshev–Gauss–Labatto (CGL) nodes, $x_j = \cos(j\pi/N), \quad j = 0, 1, \ldots, N$ is denoted by P_c and defined as $P_c f(x_j) = f(x_j)$, $j = 0, 1, \ldots, N$. Let L_n be the nth degree Legendre polynomial, which satisfies the following three term recursion formula

$$L_{n+1}(x) = \frac{2n+1}{n+1} x L_n(x) - \frac{n}{n+1} L_{n-1}(x), \quad n \geqslant 1,$$

where $L_0(x) = 1$ and $L_1(x) = x$.

$$(L_n, L_m) = \left(n + \frac{1}{2} \right)^{-1} \delta_{nm}.$$

The discrete inner product and norm based on Legendre–Gauss–Lobatto points are defined as

$$(\Psi, \Phi)_N = \sum_{j=0}^{N} \Psi(x_j)\Phi(x_j)\omega_j, \quad \|\Psi\| = (\Psi, \Psi)_N^{1/2}.$$

We state some Lemmas with out proof, which will be useful for the proof of error estimates in Sections 3 and 4.

Lemma 2.1 ([22]). *Assume that $\Psi \in H_p^m(\Omega)$, for any $0 \leqslant \mu \leqslant m$, there exists a constant C independent of Ψ and N*

$$\|\Psi - P_N\Psi\|_\mu \leqslant CN^{\mu - m}\|\Psi\|_m \quad \text{and} \quad \|P_N\Psi\|_m \leqslant C\|\Psi\|_m.$$

Lemma 2.2 ([22]). *Assume that $\Psi \in H_p^m(\Omega)$, for any $0 \leqslant \mu \leqslant m$, there exists a constant C independent of Ψ and N*

Download English Version:

https://daneshyari.com/en/article/4627358

Download Persian Version:

https://daneshyari.com/article/4627358

Daneshyari.com