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Finite-time stochastic input-to-state stability of impulsive switched stochastic nonlinear systems

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ABSTRACT

This paper mainly tends to investigate the finite-time stochastic input-to-state stability (FTSISS) problem for a class of impulsive switched stochastic nonlinear systems. An efficient lemma is firstly established to construct a generalized class $\mathcal{KL}$ function. Further, a set of Lyapunov-based sufficient conditions are derived to check the finite-time stability in probability (FTSiP) and finite-time input-to-state stability (FTISS) properties of switched stochastic nonlinear systems with or without impulses. In particular, some useful results in favor of simulations are obtained in terms of average dwell-time technique. Finally, a numerical example is provided to illustrate the effectiveness of proposed results.

1. Introduction

In the field of engineering, stochastic optimization [1–3] is coming to play an increasingly important role in the analysis and control of stochastic systems. Switched systems are referred to a family of continuous-time or discrete-time subsystems and a switching signal that determines the switching mode between them [4,5]. For the related topics within the uncertain systems framework in the current work, one can see [6,7] for some reference. In [8–10], the filtering and control problems are studied for Markov jump systems. Further, in some practical systems, such as physical, engineering, biological and information science systems, impulsive dynamical behaviors inevitably exist due to abrupt changes at certain instants during the dynamical process. In recent years, impulsive switched systems have drawn much attention and many useful conclusions have been obtained for determined systems (see e.g., [11,12]), and for stochastic case, (see e.g., [13]) for some details.

As is well known, how to characterize the effects of external inputs on state property is a meaningful topic when one investigates the stability. The concept of input-to-state stability (ISS), introduced by Sontag [14,15], is proved useful in this regard. Recently, the ISS analysis is addressed in [16] for switched systems. Finite-time stability (FTS), as an important field of stability property, are broadly investigated; which can be classified into two categories. One can be described as the system state does not exceed a certain bound during a specified time interval [17]. The other can be defined as the system state reaches the system equilibrium in a finite time [18]. Here, we shall focus our attention upon the latter case. As a composite concept, finite-time input-to-state stability (FTISS) has attracted much attention and some efficient results have been derived [19].

In [20], it is concerned with checking ISS and integral-ISS (iISS) properties for nonlinear impulsive systems. A novel impulsive system approach is designed to analyze the ISS property of networked control systems [21]. Some useful results are presented in [22] to estimate the globally asymptotical stability for switched stochastic nonlinear systems. It should be pointed

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out that FTISS stability analysis is not considered for nonlinear switched systems by now. However, it is obvious that the FTS is more meaningful than the GAS in some sense, which motivates our research.

In this paper, we pay attention to the FTSiP and FTSISS problems concerning the switched stochastic nonlinear systems; which are rarely studied in the existing references. At first, the definitions of FTSiP and FTSISS are both introduced in the forms of generalized class $\mathcal{KL}$ functions. Then a comparison principle considering the finite-time convergence is extended to the stochastic systems, which shows how to construct a generalized class $\mathcal{KL}$ function explicitly. Based on that, some sufficient conditions are provided to verify the properties of FTSiP and FTISS for switched stochastic nonlinear systems utilizing Lyapunov function approach. Meanwhile, average dwell-time technique is applied to design appropriate switching rule to obtain the desired finite-time stability. In addition, we generalize our research to the impulsive switched stochastic nonlinear systems, which is proved to be hard in the sense that choosing an appropriate impulsive function to satisfy certain Lyapunov function conditions. Further, average dwell-time approach is again used to derive some useful results in favor of simulation. In the end, we illustrate the efficiency of the results derived in this paper through one example.

To summarize, this paper mainly has the following contribution: (1) This paper considers the stochastic switched impulsive nonlinear systems, which are a kind of complex hybrid systems having just caused a little research up to now. The complexity introduced by switching rule and stochastic disturbance makes it difficult to check the input-to-state stability property but worthwhile in the sense that it may bring much attention to this field. (2) The FTSISS property for stochastic switched systems is a new and compound concept which combines finite-time stability behavior with input-to-state stability property. Different from existing works in this respect, this paper provides an efficient comparison principal for stochastic systems, which makes it possible to estimate the finite-time stability by a constructive method. Particularly, it should be noted that the introduction of positive constant c plays a critical role in constructing a generalized class $\mathcal{KL}$ function to ensure the finite time convergence performance. (3) This paper discusses the impulsive effects at switching instants, making the study of FTSISS property more challenging. Though it does not show clearly how to derive some conditions dependent on impulsive function, this paper actually deal with the problem in terms of the multiple Lyapunov functions method.

2. Problem formulation and preliminaries

Throughout this paper, let $\mathbb{R}^n$ be the n-dimensional Euclidean space; $\mathbb{R}^+$ is the set of all nonnegative real numbers; $\|\cdot\|$ denotes the standard Euclidean norm for vectors; $\mathcal{L}_\infty^m$ stands for the set of all the measurable and locally essentially bounded vector function defined in $\mathbb{R}^m$; $\mathbb{E}[\cdot]$ means the expectation of a stochastic process; $\mathcal{N} = \{1, 2, \ldots, N\}$ is a discrete index set, where N is a finite positive integer.

Consider the following n-dimensional stochastic nonlinear system

$$dx = f(t, x, u)dt + g(t, x, u)dw, \quad t \geqslant t_0, \tag{1}$$

where $x \in \mathbb{R}^n$ and $u \in \mathcal{L}_\infty^m$ are system state and input respectively; $w(t)$ is an r-dimensional Brownian motion defined on the complete probability space $(\Omega, \mathcal{F}, \{\mathcal{F}_t\}_{t \geqslant t_0}, P)$, with Ω being a sample space, $\mathcal{F}$ being a σ-field, $\{\mathcal{F}_t\}_{t \geqslant t_0}$ being a filtration and P being a probability measure; initial state $x_0 \in \mathbb{R}^n$. $f : [t_0, \infty) \times \mathbb{R}^n \times \mathbb{R}^m \to \mathbb{R}^n$, and $g : [t_0, \infty) \times \mathbb{R}^n \times \mathbb{R}^m \to \mathbb{R}^{n \times r}$ are both continuous with respect to t, x, u, uniformly locally Lipschitz with respect to u and satisfy $f(\cdot, 0, 0) = 0, g(\cdot, 0, 0) = 0$.

For convenience, we first introduce some definitions as follow.

Definition 1 [23]. A function $\gamma : \mathbb{R}^+ \to \mathbb{R}^+$ is said to be a generalized class $\mathcal{K}$ function if it is continuous with $\gamma(0) = 0$, and satisfies

$$\begin{cases} \gamma(s_1) > \gamma(s_2), & \gamma(s_1) \neq 0, \\ \gamma(s_1) = \gamma(s_2) = 0, & \gamma(s_1) = 0 \end{cases} \tag{2}$$

for all $s_1 > s_2 \geqslant 0$.

Definition 2. A function $\beta : \mathbb{R}^+ \times \mathbb{R}^+ \to \mathbb{R}^+$ is a generalized class $\mathcal{KL}$ function if, for each fixed $t \geqslant 0, \beta(s, t)$ is a generalized class $\mathcal{K}$ function in s, and for each fixed $s \geqslant 0$, it decreases to zero as $t \to T$ for some finite constant $T > 0$.

Remark 1. Note that conventional class $\mathcal{K}$ functions belong to the generalized class $\mathcal{K}$ functions, which is not the same case with class $\mathcal{KL}$ functions.

Definition 3. The equilibrium $x = 0$ of system (1) with $u(t) = 0$ is FTSiP if, for any $\epsilon > 0$, there exists a generalized class $\mathcal{KL}$ function β such that

$$P\{\|x(t)\| \leqslant \beta(\|x_0\|, t - t_0)\} \geqslant 1 - \epsilon, \quad t \geqslant t_0 \tag{3}$$

holds for any initial state $x(t_0) = x_0$.