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Stability of complex-valued impulsive system with delay

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ABSTRACT

Since many systems such as quantum system, complex-valued neural network and Lorenz system are complex-valued differential system, in this paper, the stability of complex-valued impulsive system with delay is addressed. By taking advantage of Lyapunov function in complex field, some new criteria for global exponential stability are established, which not only generalize some known results in literature but also greatly reduce the complexity of analysis and computation. As an application, a new impulsive controller for complex-valued Lü system with delay is designed.

1. Introduction

Since many evolution processes, optimal control models in economics, stimulated neural networks, frequency-modulated system and some motions of missiles or aircrafts are characterized by impulsive dynamical behavior, impulsive system is playing an increasingly important role. Over the last 10 years, there are many researchers who have studied many properties of impulsive system, due to their theoretical and practical significance, for example [1–10] and the references therein.

However, the common setting adopted in aforementioned works is always in real field, namely, the objectives of study are real-valued differential system. The objective of study in this paper is complex-valued differential system. Complex-valued differential system has also many potential applications in science and engineering. For example, quantum system, which is one of the foci of ongoing research [11–14], is a classical complex-valued differential system. Another important example of complex-valued differential system is complex-valued neural network. Complex-valued neural networks have been found highly useful in extending the scope of applications in optoelectronics, filtering, imaging, speech synthesis, computer vision, remote sensing, quantum devices, spatio-temporal analysis of physiological neural devices and system, and artificial neural information processing [15–18]. In fact, except quantum system and complex-valued neural network, there exist many other complex-valued system equations such as Ginzburg–Landau equation [19], Orr–Sommerfeld equation [20], complex Riccati equation [21], complex Lorenz equation [22] and so on. Hence, it is meaningful and important to study the properties of complex-valued differential system.

Generally speaking, complex-valued differential system is more complex and difficult than real-valued differential system. The usual method analyzing complex-valued differential system is to separate it into real part and imaginary part, and then to recast it into a equivalent real-valued differential system, see [15,22–25] and references therein. But this method encounters two problems. One is that the dimension of the real-valued system is double that of complex-valued system, which leads to the difficulties on analysis. The other is that this method needs an explicit separation of complex-valued

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function $f(t,z)$ into its real part and imaginary part, however, this separation is not always expressible in an analytical form. An efficient way to analyze complex-valued system is to retain the complex nature of system and consider its properties on $\mathbb{C}^n$, see [14,26–29] and references therein.

To the best of our knowledge, there have been few reports about the analysis and synthesis of complex-valued impulsive system with delay. In this paper, the stability of complex-valued impulsive system with delay is addressed. By taking advantage of Lyapunov functions in complex field, new stability criteria of complex-valued impulsive system with delay are established, which not only generalizes some known results in literature but also greatly reduces the complexity of analysis and computation. Moreover a new impulsive controller for complex-valued Lü system [23] with delay is designed. Those problems are meaningful and challenging.

The remainder of the paper is organized as follows. In Section 2, the complex-valued impulsive system with delay to be dealt with are formulated and several results about the complex matrices are presented. The global exponential stability criteria of complex-valued impulsive system with delay are established in Section 3. Furthermore the impulsive control of complex-valued Lü system is discussed in Section 4. Finally, some conclusions are drawn in Section 5.

2. Notations and preliminaries

In this section, we introduce notations, definitions, and preliminary facts which are used throughout this paper.

$\mathbb{R}, \mathbb{R}_+, \mathbb{N}$ and $\mathbb{N}_0$ denote the sets of real number, nonnegative real numbers, positive integers, and nonnegative integers, respectively. Let $\mathbb{C}^n$ be the complex n-dimensional Euclidean space and $\mathbb{C}^{m\times n}$ denotes the set of $m\times n$ complex matrices. $\mathbb{H}^{n\times n}$ denotes the set of $n\times n$ Hermite matrices. The identity matrix of order n is denoted as I_n.

$\mathrm{PC}(\mathbb{I},\mathbb{C}^n)$ stands for the set of functions $\varphi:\mathbb{I}\mapsto\mathbb{C}^n$ that are continuous everywhere except at enumerable points t at which $\varphi(t^+)$ and $\varphi(t^-)$ exists such that $\varphi(t^-)=\varphi(t)$, where $\mathbb{I}\subset\mathbb{R}$ is an interval.

For $z\in\mathbb{C}^n, z^T, \bar{z}, \mathrm{Re}(z)$ and z^* are the transpose, conjugate, real part and conjugate transpose of z, respectively, the norm of z is $\|z\|=\sqrt{z^*z}$.

For $M\in\mathbb{C}^{m\times n}, M^T, \bar{M}, M^*, \lambda_{\min}(M)$ and $\lambda_{\max}(M)$ are the transpose, conjugate, conjugate transpose, minimum and maximum eigenvalue of M, respectively.

Let $\mathrm{C}([-\tau,0],\mathbb{C}^n)$ be a Banach space of continuous with the norm $\|\phi\|=\sup_{-\tau\leqslant s\leqslant 0}\|\phi(s)\|$. Denote $[\phi(t)]_\tau=([\phi_1(t)]_\tau, [\phi_2(t)]_\tau,\ldots,[\phi_n(t)]_\tau)^T, [\phi_k(t)]_\tau=\sup_{-\tau\leqslant s\leqslant 0}\{\phi_k(t+s)\}$.

Consider the following complex-valued impulsive system with variable delays defined by

$$\begin{cases}\dot{z}=Az+Bz[t-\tau(t)]+C\bar{z}+f(t,z)+g[t,z(t-\tau(t))], & t\neq t_k,\\ z(t_k^+)=\varphi_k(t_k,z(t_k)), & k\in\mathbb{N},\\ z(t_0+s)=\phi(s)\in\mathrm{PC}([-\tau,0],\mathbb{C}^n),\end{cases}\tag{1}$$

where $z\in\mathbb{C}^n$ is the state variable, $A,B,C\in\mathbb{C}^{n\times n}, f(\cdot,\cdot), g(\cdot,\cdot), \varphi_k(\cdot,\cdot):[0,+\infty)\times\mathbb{C}^n\mapsto\mathbb{C}^n$ are continuous vector-value functions with $f(t,0)\equiv 0, g(t,0)\equiv 0, \varphi_k(t,0)\equiv 0$, for all $t\in[t_0,+\infty)$ and $k\in\mathbb{N}$, and ensuring the existence and uniqueness of solutions for (1). Assume that time sequence $\{t_k\}$ satisfies

$$0\leqslant t_0<t_1<t_2<\cdots<t_k<\cdots,\quad \lim_{k\to\infty}t_k=+\infty,$$

$\sigma=\sup_{k\in\mathbb{N}_0}\{t_{k+1}-t_k\}<+\infty, \tau(t)$ is the time-delay and satisfies $0\leqslant\tau(t)\leqslant\tau$, for all $t\in[t_0,+\infty)$, where $\tau\geqslant 0$ and $\delta>1$ are constants with $\sigma>\delta\tau$.

Remark 2.1. The motivations discussing complex-valued system are chiefly as follows:

(1) Equations of many classical system except quantum system and complex valued neural network, such as Ginzburg–Landau equation [19], Orr–Sommerfeld equation [20], complex Riccati equation [21], and complex Lorenz equation [22], are considered in $\mathbb{C}^n$. Hence, it is significative and important to study the properties of complex-valued differential system.

(2) There exist differences between complex-valued and real-valued system, for example,

$$\frac{dz}{dt}=z^2+1\tag{2}$$

has equilibrium points $z=\pm\sqrt{-1}$ in $\mathbb{C}$. But (2) has not equilibrium point in $\mathbb{R}$. Another example, if we consider Ikeda-type oscillator system

$$\frac{dz}{dt}=-az(t)+b\sin z(t-\tau(t))$$

in $\mathbb{C}^n$, the nonlinear function $\sin z$ does not satisfy Lipschitz condition, since

$$\lim_{y\to+\infty}\frac{\|\sin(\sqrt{-1}y)\|}{\|\sqrt{-1}y\|}=\lim_{y\to+\infty}\frac{\|e^{-y}-e^{y}\|}{\|y\|}=+\infty.$$