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Two-stage stochastic programming problems involving multi-choice parameters

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ABSTRACT

In this paper, we propose a two-stage stochastic linear programming model considering some of the right hand side parameters of the first stage constraints as multi-choice parameters and rest of the right hand side parameters of the constraints as exponential random variables with known means. Both the randomness and multi-choiceness are simultaneously considered for the model parameters. Randomness is characterized by some random variables with its distribution and multi-choiceness is handled by using interpolating polynomials. To solve the proposed problem, first we remove the fuzziness and then for multi-choice parameters interpolating polynomials are established. After establishing the deterministic equivalent of the model, standard mathematical programming technique is applied to solve the problem. A numerical example is presented to demonstrate the usefulness of the proposed methodology.

1. Introduction

In real life decision making problems, the input parameters of the problem under study are not known with certainty because relevant data are inexact or scarce, difficult to estimate, the system is subject to changes, etc. The uncertain parameters in the problem can often be modeled as random variables with "known" probability distribution. The occurrence of randomness in the problem parameters can be modeled as stochastic programming (SP) problem. SP is widely used in several real life problems such as, financial planning and control, supply chain management planning, airline planning (fleet assignment), water resource modeling, forestry planning, environmental planning, banking, and telecommunication.

In a conventional two-stage stochastic program, a first-stage program must be solved before all the problem parameters are known with certainty. Once the uncertainty is realized, the decision maker then solves a second stage program, known as a recourse problem, that considers the solution to the first stage program as well as the outcome of a random event. The objective is to minimize the first-stage cost plus the expected second-stage cost. The formulation of two-stage stochastic programming problems was first introduced by Dantzig [8]. Further it was developed by Beale [1] and Dantzig and Madansky [9].

The situation occurs when there exist a set of choices for a parameter, out of which only one is to be selected to optimize the objective function of a mathematical programming problem. Such type of problem is modeled as multi-choice programming problem which was introduced by Healey [11]. He presented a class of combinatorial optimization problems with a

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requirement to choose among several possible combinations as an alternative to optimize an objective function subject to a set of constraints. This type of problems occur in many practical situations, such as selecting a new car, selecting a new security personnel, employing a number of sales persons in a particular time bound in a super market, implementing a new policy for a community, media planning, multiple choice knapsack problem, sales resource allocation, multi-item scheduling, timetabling, etc. Several literatures [2–4,6,7,10] have been reported on multi-choice programming problem.

2. Stochastic programming problem

The occurrence of randomness in the model parameters can be formulated as stochastic programming (SP) within a general optimization framework. Due to the robustness, SP is widely used in many real-world decision making problems of management science, engineering, and technology. Also, it has been applied to various areas such as, manufacturing product and capacity planning, electrical generation capacity planning, financial planning and control, supply chain management, airline planning (fleet assignment), water resource modeling, forestry planning, dairy farm expansion planning, macroeconomic modeling and planning, portfolio selection, traffic management, asset liability management, etc.

Mathematically, a stochastic programming [5] problem can be stated as:

$$\min : z = \sum_{j=1}^{n} c_j x_j, \tag{2.1}$$

subject to

$$\sum_{j=1}^{n} a_{ij} x_j \leqslant b_i, \quad i = 1, 2, \ldots, m, \tag{2.2}$$

$$\sum_{j=1}^{n} r_{sj} x_j \geqslant h_s, \quad s = 1, 2, \ldots, l, \tag{2.3}$$

$$x_j \geqslant 0, \quad j = 1, 2, \ldots, n, \tag{2.4}$$

where $x_j, j = 1, 2, \ldots, n$ are the decision variables, $a_{ij}, i = 1, 2, \ldots, m, j = 1, 2, \ldots, n$ and $r_{sj}, s = 1, 2, \ldots, l, j = 1, 2, \ldots, n$ are the coefficients of the technological matrices, $c_j, j = 1, 2, \ldots, n$ are the coefficients associated with the objective function and $h_s, s = 1, 2, \ldots, l$ are the right hand side parameters of the extra constraints in the problem. Only the right hand side parameters $b_i, i = 1, 2, \ldots, m$ are considered as random variables with known means.

When some of the model input parameters are considered as multi-choice parameters, then the above model (2.1)–(2.4) can be restated as:

$$\min : z = \sum_{j=1}^{n} c_j x_j, \tag{2.5}$$

subject to

$$\sum_{j=1}^{n} a_{ij} x_j \leqslant b_i, \quad i = 1, 2, \ldots, m, \tag{2.6}$$

$$\sum_{j=1}^{n} r_{sj} x_j \geqslant \{h_s^1, h_s^2, \ldots, h_s^{k_s}\}, \quad s = 1, 2, \ldots, S, \tag{2.7}$$

$$x_j \geqslant 0, \quad j = 1, 2, \ldots, n, \tag{2.8}$$

where the right hand side parameters $h_s, s = 1, 2, \ldots, S$ are considered as multi-choice parameters.

2.1. Two-stage stochastic programming problem

Two-stage stochastic programming (TSP) is an efficient method for solving stochastic programming problems with recourse. In the standard TSP paradigm, the decision variables of an optimization problem under uncertainty are partitioned into two sets. The decision variables decided before the actual realization of the uncertain parameters are known as first stage decision variables. After the random events have exhibited themselves, further decision can be made at a certain cost known as second-stage, or recourse, variables i.e. a second-stage decision variables can be made to minimize ‘‘penalties’’ that may appear due to any infeasibility [15].

Mathematically, a two-stage stochastic programming [12,16] problem with simple recourse can be stated as: