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Existence and global asymptotic stability of positive periodic solution of delayed Cohen–Grossberg neural networks

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ABSTRACT

In this paper, a class of periodic Cohen–Grossberg neural networks with discrete and distributed time-varying delays is considered. By an extension of the Lyapunov–Krasovskii functional method, a novel criterion for the existence and uniqueness and global asymptotic stability of positive periodic solution is derived in terms of M-matrix without any restriction on uniform positiveness of the amplification functions. Comparison and illustrative examples are given to illustrate the effectiveness of the obtained results.

1. Introduction

During the past few decades, we have witnessed an increased interest in the study of dynamics behavior of neural network models due to their potential and successful applications in many fields such as signal processing, pattern recognition, associative memory, parallel computing and solving optimization problems, and so on [1–4]. In most of the practical applications, it is of prime importance to ensure that the designed neural networks be stable. On the other hand, time delays unavoidably exist in most application networks and often become a source of oscillation, divergence, instability or bad performance [5–9]. Therefore, many researchers from mathematics and engineering communities have devoted significant efforts to study the stability and control of delayed neural network models (see [10–22] and the references therein).

Among the various models of neural networks which have been extensively studied and applied, the Cohen–Grossberg neural networks (CGNNs), first proposed and studied by Cohen and Grossberg [23], have received considerable attention from researchers both in theoretical and in practical applications (see, for example, [24–34] and the references therein). It was pointed out [24,25,30–33] that CGNNs include a range of well-known ecological models and neural networks such as the Lotka–Volterra system [35], the bidirectional associative memory (BAM) neural networks [20] and the Hopfield neural networks (HNNs) [3,17,22]. Many interesting results on the stability of equilibrium point for the CGNNs with constant coefficients have been reported in the literature [24–27,30,33,34]. However, it is well known that, non-autonomous phenomena often occur in many realistic systems. Particularly, when we consider a long-term dynamical behavior of the system, the parameters of the system usually will change along with time [36]. Furthermore, the properties of periodic oscillatory solutions represented in various applications such as memory and storage patterns play an important role in the dynamical behavior for non-autonomous neural networks. In addition, an equilibrium point is treated as a special periodic solution. Therefore, it is important and interesting to study the asymptotic behavior of periodic/almost periodic solutions of the non-autonomous CGNNs [36–40].

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Such applications rely not only on the existence and/or on the uniqueness of the equilibrium point but also on the asymptotic behavior such as stability, boundedness, periodicity, oscillatory etc. Many efforts have been devoted to investigate the stability, one of the major problems encountered in design and applications of neural networks, and many important results have been proposed in terms of linear matrix inequalities, M-matrices by using Lyapunov–Krasovskii functional method, Halanay inequality, or fixed point theorem technique [24,28,30,34,36,41]. However, in most of the existing results in the literature, it has been assumed directly or indirectly that the amplification functions are uniformly positive, or in another words, they are assumed to be greater than a positive constant (see, for example, [27,28,30,33,34,40,41]). This restriction usually rules out the Lotka–Volterra system [39]. Moreover, it is difficult to prove the existence and global convergence of periodic solutions for the CGNNs without uniform positiveness of the amplification functions. On the other hand, as a model of competitive neural networks, the CGNNs can be used to describe the dynamics of interacting populations. In addition, many realistic applications are involving positive solutions of such systems. Therefore, it is of interest and importance to study global asymptotic behavior of positive periodic solutions for non-autonomous CGNNs. So far, there have been few papers dealing with the existence and global asymptotic stability of periodic positive solution for non-autonomous CGNNs without the uniform positiveness assumption. Among them and most related to the present paper, we refer the reader to the paper [39] where the author developed the Lyapunov method for a class of periodic CGNNs with constant delays and autonomous amplification functions without assuming the uniform positivity of these functions.

Since a neural network usually has a spatial nature due to the presence of an amount of parallel pathways of a variety of axon sizes and lengths, it is desirable to model them by introducing continuous distributed delays over a certain duration of time such that the distant past has less influence than the recent behavior of the state [8,24]. So far, both discrete time-varying delays and distributed delays have been widely accepted as important parameters associated with neural network models. Extensive attention has been focused on the dynamics behavior of neural networks with mixed (that is both discrete and distributed) time-varying delays [12,13,15,16,22,24,41–43].

Motivated by the aforementioned works, in this paper we consider a more general class of non-autonomous periodic CGNNs with discrete and distributed time-varying delays and non-autonomous amplification functions. First, we present a modified proof of positivity of solutions using the technique employed in [39]. Then, by constructing an improved Lyapunov-like functional and by adopting the terminology in [39], a novel criterion for the existence and global asymptotic stability of positive periodic solutions of the system is derived in terms of M-matrix without any restriction on the uniform positiveness of the amplification functions. Our conditions can also be applied to autonomous systems and the Lotka–Volterra systems with time-varying delays. Furthermore, the results obtained in this paper cover those results in [39] when applying our conditions to the CGNN model considered in [39] as a special case.

The rest of this paper is organized as follows. Section 2 presents notations, definitions and some auxiliary technical lemmas which will be used in the proof of our main results. In Section 3, new conditions for the existence and global asymptotic stability of positive periodic solutions of the system are derived in terms of M-matrix. Comparison and illustrative examples are given in Section 4. The paper ends with a conclusion and cited references.

2. Preliminaries

Notations: For a given natural numer n, we denote $\underline{n} := \{1,2,\ldots,n\}$. R^+ is the set of non-negative real numbers; R^n denotes the n-dimensional space with the vector norm $\|x\| = \sum_{i=1}^n |x_i|$; $R^{m\times n}$ stands for the set of all $m\times n$-matrices with entries in R. Inequalities between two real vectors will be understood componentwise; i.e. for $u=(u_i)$ and $v=(v_i)$ in R^n, we write $u \geqslant v$ iff $u_i \geqslant v_i,\ \forall i\in\underline{n}$; if $u_i > v_i,\ \forall i \in \underline{n}$, then we will write $u \gg v$ instead of $u>v$; similar notation is adopted for matrices; $R^n_+ = \{x\in R^n : x\geqslant 0\}$ is the non-negative orthant. $C([a,b],\ R^n)$ denotes the set of all R^n-valued continuous functions on interval $[a,b]$ and $C^+([a,b],\ R^n)$ stands for the positive subset of $C([a,b],\ R^n)$, i.e. $\varphi\in C^+([a,b],\ R^n)$ iff $\varphi\in C([a,b],\ R^n)$ and $\varphi(t)\gg 0,\ \forall t\in[a,b]$. $x_t=\{x(t+s): -h\leqslant s\leqslant 0\}$ denotes the segment of the trajectory $x(t)$ on $[t-h,t],\ h>0$, with its norm $\|x_t\| = \sup_{s\in[-h,0]}\|x(t+s)\|$.

Consider the following Cohen–Grossberg neural networks with mixed time-varying delays

$$\begin{aligned} \dot{x}_i(t) =\ & a_i(t,x_i(t))\left[-b_i(t,x_i(t)) + \sum_{j=1}^n w_{ij}(t)f_j(x_j(t)) + \sum_{j=1}^n v_{ij}(t)g_j(x_j(t-\tau_{ij}(t))) + \sum_{j=1}^n q_{ij}(t)\int_{-h_{ij}(t)}^0 K_{ij}(s)p_j(x_j(t+s))ds + I_i(t)\right], \\ & t\geqslant 0,\ i\in\underline{n},\ x_i(t) = \varphi_i(t),\ t\in[-\overline{h},0],\ i\in\underline{n}, \end{aligned} \tag{1}$$

where n is the number of neurons, $x_i(t)$ is the state variable (potential or voltage) of the ith neuron at time t, $f_j(.),\ g_j(.)$ and $p_j(.)$ are activation functions, $b_i(t,x_i)$ are self-inhibition terms, and $a_i(t,x_i)$ are amplification functions, $w_{ij}(t),\ v_{ij}(t)$ and $q_{ij}(t)$ are connection weights and $K_{ij}(s)$ are the kernels of distributed state, $I_i(t)$ are external inputs, $\tau_{ij}(t)$ and $h_{ij}(t)$ are transmission time-varying delays which are continuous and satisfying the following well-studied conditions [21,28,33]

$$\begin{aligned} & 0\leqslant \tau_{ij}(t)\leqslant \overline{\tau}_{ij}, \quad \dot{\tau}_{ij}(t)\leqslant \mu, \quad 0\leqslant h_{ij}(t)\leqslant \overline{h}_{ij}, \\ & \overline{h} = \max\{\overline{\tau}_{ij}, \overline{h}_{ij}\}, \quad i,j\in\underline{n}, \end{aligned} \tag{2}$$

where $0\leqslant \mu < 1$ is a known constant.