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Distributed consensus for sampled-data control multi-agent systems with missing control inputs

Wentao Zhang, Yang Liu *

College of Mathematics, Physics and Information Engineering, Zhejiang Normal University, Jinhua 321004, China

A R T I C L E I N F O

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A B S T R A C T

This paper is concerned about the distributed consensus problem for general sampled-data control multi-agent systems without or with a leader under control inputs missing over some sampling intervals. For these two cases, a distributed adaptive dynamical consensus algorithm is proposed based only on the relative information of network structure. Under the assumption that the dynamical network contains a directed spanning tree, some sufficient consensus conditions are induced under a switched model. These obtained consensus criteria are independent of dynamical network topology, namely, they are valid for both undirected and directed topology. Besides, they do not rely on the spectra and the eigenvalue of Laplacian matrix. Furthermore, some quantitative relations included in dynamical systems, such as sampling period, admissible control inputs missing rate and exponential reaching rate of the consensus performance, are established. Finally, simulation examples are given to show the effectiveness of the proposed results.

1. Introduction

Cooperative behavior in a family of networks of autonomous mobile nodes (agents or particles) has drawn much attention due to its widespread applications, such as robotic teams, unmanned air vehicle (UAV) formation, control theory, engineering, biology, see [1–13] and the references therein. Especially, consensus is one of the most representative cooperative behavior [14–22]. Obviously, this is a kind of collective behavior. The relations between collective behavior and evolutionary games have been well explained in [23], by which we can well understand the complexity arising in such systems appearing in nature and society. The study of consensus problem has a long history and is prevalent, such as emerge under the bargain for consensus game [24], and some other similar original relevant research, for example, [25,26]. The main idea of consensus is that each agent shares local information only with its neighbors while the whole dynamic of agents can coordinate so as to achieve a certain global criterion of common interest based on a distributed protocol (strategy or rule). And the importance of interaction networks on coevolutionary rules has been pointed out in [27]. Moreover, recent interesting advances on top of structured population, such as complex networks and coevolutionary model has been presented in [28].

Many interesting consensus criteria were obtained in literature. For example, [29] provided a theoretical framework for analysis of consensus protocols for multi-agent systems. Distributed observe-type consensus algorithm was proposed to solve the consensus or synchronization problem based only on the relative output measurements in [18,30]. Similarly, in [21], consensus criteria for second-order nonlinear multi-agent systems were obtained by using distributed adaptive

* Corresponding author.
E-mail address: liuyang4740@gmail.com (Y. Liu).

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protocol. An interesting feature of these obtained consensus results is that the coupling weights are time-varying, which are distributed because only local information of each agent and neighbors are used, see [18,21,22,31] for references therein. While in [6,14,20,32], the consensus problem has also been studied, while the coupling weights are all equivalent to given constants.

On the other hand, sampled-data control is another effective method for networked control systems because of its low cost, flexibility, ease of maintenance, see [33–35] for references therein. Many pioneering consensus conditions were derived by this approach, such as [14,15]. In [14], under the assumption that the relative velocity states of agents were unavailable and the communication topologies contained a directed spanning tree, some necessary and sufficient consensus conditions were obtained by sampled-data control. In addition, control inputs missing is inevitable in many real-world applications for the limitations of sensing ranges, the failure of physical, the unreliability of communication channels, etc. Some consensus conditions were obtained with control input missing under the directed or undirected topologies in [15,17]. However, these induced consensus results depend on the spectra and the eigenvalue of Laplacian matrix, which implies that the global information of the systems should be known. What is more, it is a huge work for computation when the network is large-scale.

Motivated by above discuss, in this paper, distributed consensus problem for general sampled-data control multi-agent systems with missing control inputs is investigated. First of all, a distributed adaptive dynamical consensus algorithm is designed based only on the local relative information of the systems. The limitation that consensus criteria rely on the spectra or eigenvalue of the Laplacian matrix is moved, which played a key role for the consensus reaching in [15–17]. Furthermore, based on the proposed consensus protocol, the obtained consensus conditions are also valid for both directed and undirected topologies. Moreover, the relations between consensus performance of multi-agent systems (exponential decay and increasing rate), control inputs missing rate and sampling period are established.

The rest of the paper is organized as follows. In Section 2, some preliminaries on graph theory and model formulation are given. The main results about distributed consensus in sampled-data control multi-agent dynamical systems with missing control inputs for both cases without and with a leader are presented in Sections 3 and 4, respectively. In Section 5, numerical examples are given to illustrate the theoretical analysis. Conclusions are finally drawn in Section 6.

Throughout this paper, $I_n,\ 0_n$ denote the n-dimensional identity matrix and n-dimensional zero matrix, respectively. $\mathbf{I} \in R^n$ is a vector with all entries be one. The superscript "$'$" stands for matrix transposition if no confusion arises. $diag(a_1, a_2, \ldots, a_N)$ indicates the diagonal matrix with diagonal entries $a_1, a_2, \ldots, a_N$. Notation $\|\cdot\|$ represents the Euclidean norm. In particularly, data missing is referred to as the control input missing.

2. Model description and some preliminaries

2.1. Mathematical preliminaries

In this section, some basic terminologies and results from algebraic graph theory are first introduced. Let $\mathcal{G} = (\mathcal{V}, \mathcal{E}, G)$ be a weighted directed or undirected graph of order N, with $\mathcal{V} = \{v_1, v_2, \ldots, v_N\}$ representing the set of nodes. $\mathcal{E} \in \mathcal{V} \times \mathcal{V}$ is the set of directed or undirected edges, and $G = (g_{ij})_{N\times N}$ denotes the underlying weighted adjacency matrix with nonnegative elements. If node i can receive the information from node j, then $g_{ij} > 0,\ j \neq i$; otherwise, $g_{ij} = 0$. A directed graph contains a directed spanning tree if one of its subgraphs is a directed spanning tree. The Laplacian matrix $L = (l_{ij})_{N\times N}$ is defined by $l_{ii} = -\sum_{j=1, j\neq i}^{N} l_{ij},\ l_{ij} = -g_{ij},\ i \neq j$, which ensures the property that $\sum_{j=1}^{N} l_{ij} = 0$.

Definition 1 [19]. For any $t > t_0 \geqslant 0$, let $N_{(t_0,t)}$ denote the number of switching of $\sigma(t)$ over $[t_0, t]$. If $N_{(t_0,t)} \leqslant \frac{t-t_0}{T_a} + N_0$ holds for $T_a > 0$ and $N_0 \geqslant 0$, then T_a is called the average dwell time and N_0 the chatter bound.

Without loss of generality, we select $N_0 = 0$ and I stands for the identity matrix I_N throughout this paper.

2.2. Dynamical systems presentation

Consider a group of N identical nodes (agents) with general linear dynamics. The dynamics of the ith node (agent) is represented by

$$\dot{x}_i(t) = Ax_i(t) + F\sum_{j\in\mathcal{N}_i} h(t) l_{ij}(x_j(t) - x_i(t)) + B_1 U_i(t), \quad i = 1, 2, \ldots, N, \tag{1}$$

where $U_i(t) = Kx_i(t_k),\ t \in [t_k, t_{k+1}),\ k = 0, 1, \ldots,\ \ h(t)$ denotes the time-varying coupling gain for node (agent) i. $x_i(t) \in R^n,\ U_i(t) \in R^p$ and $\mathcal{N}_i$, respectively, are the state, the control input and the neighbors of node i. $A,\ B_1,\ F$ and K are constant matrices with compatible dimensions. Furthermore, let $t_{k+1} - t_k = T$, i.e., the sampled-data is period.

The purpose of this note is to search the relations between control inputs missing rate, sampled-data period and distributed consensus performance under the control inputs missing effect such that the consensus of (1) is reached. The dynamical equation (1) can be rewritten into a compact form as follows

$$\begin{aligned} &\dot{x}(t) = (I\otimes A - h(t)L\otimes F)x(t) + (I\otimes B_1K)x(t_k), \\ &t \in [t_k, t_{k+1}), \quad k = 0, 1, \ldots \end{aligned} \tag{2}$$