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Diversified portfolios with different entropy measures

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ABSTRACT

One of the major issues for Markowitz mean–variance model is the errors in estimations cause "corner solutions" and low diversity in the portfolio. In this paper, we compare the mean–variance efficiency, realized portfolio values, and diversity of the models incorporating different entropy measures by applying multiple criteria method. Differing from previous studies, we evaluate twenty-three portfolio over-time rebalancing strategies with considering short-sales and various transaction costs in asset diversification. Using the data of the most liquid stocks in Taiwan's market, our finding shows that the models with Yager's entropy yield higher performance because they respond to the change in market by reallocating assets more effectively than those with Shannon's entropy and with the minimax disparity model. Furthermore, including entropy in models enhances diversity of the portfolios and makes asset allocation more feasible than the models without incorporating entropy.

1. Introduction

The Markowitz [1] mean–variance (MV) model has been widely applied both in academia and in the real-world for portfolio selection. Numerous studies have analyzed the effectiveness of diversification in investment strategies based on its risk-return trade-off relation [2]. However, such optimal portfolios are found problematic to be exercised in practice due to the error in estimation of the moments of asset returns. The major issues for asset management are (1) the MV portfolios are often extremely concentrated in a limited number of assets, and (2) the out-of-sample performances generated are poor. The estimation errors may bias the optimal portfolio weights and may cause those portfolios to generate "corner solutions" that involve infeasible asset allocations [3]. There has been extensive research on reducing statistical errors in the mean and variance–covariance matrixes due to the bias caused by the use of unpredicted information [4,5]. Therefore the statistics generated by the entropies provide additional information in forming optimal portfolios, particularly to increase asset diversification by reducing errors in estimating associated parameters. The low diversity of the MV portfolio may result in loss while some of the invested assets experience unexpected gains [4,6–8]. In this paper, we evaluate the performance of the portfolio selections incorporating different entropy measures by applying multiple criteria method. To improve the feasibility of models, we consider the impact of short-sale constraints and transaction costs on portfolios.

Portfolio diversification implies that the idiosyncratic risk can be decreased to zero as the assets included in the investment increase [9]. Therefore, the purpose of a diversified portfolio is to invest in as many mean–variance efficient assets as

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possible. One of the major issues of realization of a conventional portfolio selection model is low diversity or the corner solutions. Chiou et al. [10] suggest that setting the upper bound of weight reduces the uncertainty but sacrifice the performance of diversified portfolio. DeMiguel et al. [8] show that a naïve equally-weighted portfolio demonstrates higher out-of-sample diversification performance than the optimal MV model. In our study, we apply various entropy measures, including Shannon's, Yager's entropy, minimax disparity model, as theoretical foundation to model portfolio rebalancing and further to compare the naïve portfolio suggested by DeMiguel et al. [8].

Trading costs play a crucial role in constructing realistic strategies that can be applied in the financial industry [6,11–15]. In our study, we incorporate real brokerage fee and taxes and apply multiple criteria decision making (MCDM) to maximize the portfolio realized return. The technical difficulty for computing realization portfolio values with considering trading costs in this study is higher than previous literature using return and mean–variance efficiency. This paper contributes to literature by applying advanced methodology to evaluate effectiveness of various optimal portfolios in the real world.

The entropy serves an alternative measure of uncertainty in information theory, econometrics, and finance [16]. Among the measures, the Jaynes [17] selection criterion is referred as the maximum entropy criterion [18], which is a rule to assign numerical values to probability in circumstances that certain partial information is available [19]. Assuming the given moments represent known information, the maximum entropy (ME) principle chooses the one having maximum entropy or equivalently the most uncertain distribution [20].

Entropy is useful to model a least biased distribution from the partial information represented by certain moment restrictions [21]. Shannon [16] proposes a non-linear model to estimate entropy. Yager [22] later applies the maximum entropy principle. Wang and Parkan [23] proposed a linear entropy model, the minimax disparity model, which minimizes the maximum difference between each pair of weights. Lutgens and Schotman [24] use a mini-max portfolio strategy to analyze the decisions and performance of a robust decision maker. Philippatos and Wilson [25] first apply entropy as a measurement of the uncertainty in portfolio selection. They suggest that entropy is more general than variance as a measure of risk, because it is free from reliance on the assumption of symmetric probability distributions and can be computed from non-metric data [26]. Jiang et al. [18] present a maximum entropy portfolio model for large scale portfolio problems. Though the above models are free of any assumption regarding the distribution, they are static and fail to include rebalancing process.

The weights of portfolio obtained through the maximum entropy (ME) approach are in the form of "probabilities," therefore the weights are non-negative. However, allowing short-sale gives investors flexibility in managing their portfolios, particularly during the bearish market or for managing hedge funds [4]. To our best knowledge, previous studies related to entropy do not consider portfolio short-sale in models. White [27] points out that short-sale can be used to lower investment risks and to improve a portfolio's risk-return trade-off [28]. Investors also can use short-sale to engage portfolio arbitrage [29]. In our study, we compare the realized portfolio values of twenty-three models and their change in portfolio weights [30].

To generate realistic results, we rebalance the optimal portfolios that (1) allow short selling assets, (2) apply various entropy measures, and (3) consider various transaction costs. Our finding shows that the models with Yager's entropy yield higher portfolio value than those with Shannon's entropy and those with the minimax disparity model. This is because the models of Yager's entropy respond the change in market by reallocating assets in the portfolio more effectively. In addition, including entropy in models enhance diversity of the portfolios. Given the fact that our portfolios with entropy measures are less subject to the variation in sample, these portfolios are robust [24].

The remainder of this paper is organized as follows. Section 2 introduces the entropy models and portfolio selection models. In Section 3, we present portfolio rebalancing method. Section 4 reports the numerical results. The conclusions are presented in Section 5.

2. Entropy models and portfolio selection models

We first introduce Shannon's entropy, Yager's entropy and the mini-max disparity model in this section. We then present portfolio selection models.

2.1. Shannon's entropy

Shannon's entropy [16] is first developed to solve communication problems and later is applied in finance to measure the amount of information given by observing the market. Simonelli [31] points out that Shannon's entropy is more useful in constructing a portfolio than using variance or other deviation measures. Using Shannon's entropy in portfolio selection can diversify the allocation on various assets, while meeting the requirement of investors.

The following is Shannon's entropy:

$$H = -\sum_{i=1}^{n} w_i \ln w_i, \;\; \sum_{i=1}^{n} w_i = 1, \quad i = 1, 2, \ldots, n, \tag{1}$$

w_i the weight of security i (the probability of outcome i); n the number of invested securities (the number of states).

H has the maximum value, while $w_i = \frac{1}{n}$; the larger the H, the more information is gained by the observations. The other extreme case occurs when $w_i = 1$ for one i, and =0 for the rest, then $H = 0$. Therefore, Shannon's entropy provides a measure