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Mathematical modeling of electric power flow and the minimization of power losses on transmission lines

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ABSTRACT

The importance of electric power in today's world cannot be overemphasized, for it is the key energy source for industrial, commercial and domestic activities. Its availability in the right quantity is essential to advancement of civilization. Electrical energy produced at power stations is transmitted to load centres from where it is distributed to the consumers through the use of transmission lines run from one place to another. As a result of the physical properties of the transmission medium, some of the transmitted power are lost to the surroundings. The power losses could take off a sizeable portion of the transmitted power since the transmission lines usually span a long distance, sometimes several hundred kilometers. The overall effect of power losses on the system is a reduction in the quantity of power available to the consumers. As such, adequate measures must be put in place to reduce power losses to the barest minimum. Thus, in this paper, we developed a mathematical model for determining power losses over typical transmission lines, as the resultant effect of ohmic and corona power losses, taking into cognizance the flow of current and voltage along the lines. Application of the classical optimization technique aided the formulation of an optimal strategy for minimization of power losses on transmission lines. With the aid of the new models it is possible to determine current and voltage along the transmission lines. In addition, we note that the analytical method does not involve any design or construction and so is less expensive than other models reported in the literature. Hence, the goal of this paper is to address a very well-known engineering problem – reducing the power losses on transmission lines to the barest minimum.

1. Introduction

Energy is a basic necessity for the development of any nation. Although, there are different forms of energy, the most important of them is electrical energy. A modern and civilized society is so much dependent on the use of electrical energy because it has been the most powerful vehicle for facilitating economic, industrial and social developments [21].

The ever increasing use of electrical energy for industrial, domestic and commercial purposes necessitated the bulk production of electrical energy. This bulk production is achieved with the help of suitable power production stations which

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are generally referred to as electric power generating stations or electric power plants. A generating station usually employs a prime mover coupled with an alternator to produce electric power [18].

Electrical energy produced at power stations is transmitted to load centres from where it is distributed to the consumers [10]. The transmission and distribution subsystems are very important to the electric power system, because without these subsystems the generated power cannot get to the load centres not to talk of getting finally to the consumers.

The connection between the power stations and the load centres is effected with the use of transmission lines, usually conductors run from one place to another and supported on towers. However, the arrangement of the power system places the transmission subsystem in a critical position since it is only the quantum of energy delivered to the distribution subsystem that will be said to be available for consumption in the system. For this reason, what happens in the transmission subsystem demand a careful examination. In designing extra high voltage single-circuit, Sakhavati et al. [26] highlight the importance of transmission lines.

The efficiency of the transmission component of the electric power system is known to be hampered by a number of problems, especially in third-world countries. The major problems identified in [14] include application of inappropriate technology, inadequacy of materials, equipment and man-power, among others. Even in developed countries, efficiency of the transmission subsystem has been shown to depend on the topology and type of the electric power grid adopted, see Albert et al. [3], Kinney et al. [15] and Han and Ding [12].

From the physics of electric power transmission, when a conductor is subjected to electric power (or voltage), electric current flows in the medium. Resistance to the flow produces heat (thermal energy) which is dissipated to the surroundings. This power loss is referred to as ohmic loss [28]. Furthermore, if the applied voltage exceeds a critical level, another type of power loss, called the corona effect [9] occurs. The power losses accumulate as the induced current flows and the corona effect propagate along the transmission lines. The power losses could take off a sizeable portion of the transmitted power since transmission lines usually span a long distance, sometimes several hundred kilometers [11]. The overall effect of power losses on the system is a reduction in the quantity of power available to the consumers. Thus, adequate measures must be put in place to reduce power losses to the barest minimum.

A lot of research works have been undertaken on minimization of electric power losses, out of which we cite a few. Ramesh et al. [23] looked at minimization of power losses in distribution networks by using feeder restructuring, implementation of distributed generation and capacitor placement method. Rugthaicharoencheep and Sirisumrannukul [25] employed the use of feeder reconfiguration for loss reduction in distribution system with distributed generators by Tabu Search. Numphetch et al. [19] worked on loss minimization using optimal power flow based on swarm intelligences while Abddullah et al. [1] studied transmission loss minimization and power installation cost using evolutionary computation for improvement of voltage stability. In addition to active power losses, series reactive power losses of transmission system were also considered as one of the multiple objectives, and Zakariya [30] made a comparison between the corona power loss associated with HVDC (High Voltage Direct Current) transmission lines and the ohmic power loss.

In most of the above research works, much emphasis has been on reduction of losses using feeder reconfiguration, implementation of distributed generation and capacitor placement which are capital intensive. A more general approach would be to blend the art of mathematical modeling with the mathematical precision of the classical optimization technique for minimization of power losses.

In a previous effort (see Oke and Bamigbola [20]), the classical optimization technique was applied for minimization of the power losses function without taking into consideration the fact that the voltage varies along the transmission lines. In this paper, we developed a mathematical model for determining power losses over typical transmission lines as the resultant effect of ohmic and corona losses but taking into cognizance the flow of current and voltage along the lines. Application of the classical optimization technique aided the formulation of an optimal strategy for minimizing power losses on transmission lines.

With the aid of the new models it is possible to determine current and voltage along transmission lines. In addition, the present approach is economical in terms of cost and effort as it does not involve any design or construction of electrical appliances.

2. Power flow on transmission lines

In this section, we derive the expressions which voltage and current must satisfy on uniform transmission lines. A real transmission line will have some series resistance associated with power losses in the conductor [4]. There may also be some shunt conductance if the insulating material holding two conductors has some leakage current. Therefore, resistance and conductance are responsible for power losses on transmission lines [7]. To this end, we formulate a model for a lossy transmission line where the effect of the series resistance (R) and shunt conductance (G) are taken care of.

2.1. Model formulation

Herein, we are interested in determining the extent to which current and voltage outputs differ from their input values over an elemental portion of the transmission line. As such, we consider an equivalent circuit of a transmission line of length Δx containing resistance $R\Delta x$, capacitance $C\Delta x$, inductance $L\Delta x$ and conductance $G\Delta x$ as shown in the circuit in Fig. 1 below. The circuit illustrates how power (both voltage and current) flow through the transmission medium [13].