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An epidemic model with different distributed latencies and nonlinear incidence rate

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ABSTRACT

An SEIR epidemic model with different distributed latencies and general nonlinear incidence is presented and studied. By constructing suitable Lyapunov functionals, the biologically realistic sufficient conditions for threshold dynamics are established. It is shown that the infection-free equilibrium is globally attractive when the basic reproduction number is equal to or less than one, and that the disease becomes globally attractively endemic when the basic reproduction number is larger than one. The criteria in this paper generalize and improve some previous results in the literatures.

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1. Introduction

In modeling disease transmission, the host population is often divided into disjoint classes of susceptible, exposed, infective and recovered individuals, with numbers at time t denoted by $S(t),\ E(t),\ I(t),\ R(t)$ respectively. In mathematical epidemiology models, the incidence rate function can have a crucial role for modeling of epidemic dynamics.

Traditionally, it was postulated that the spread of an infectious disease in a population obeys the principle of mass action, that is, the corresponding incidence rate is bilinear with respect to the numbers of susceptible and infective individuals (see [1,9]); However, several authors argued that the real disease transmission process should have a nonlinear incidence rate (see [15,3] et al.). For this reason, infectious disease models of nonlinear incidence rates have been attracting considerable attention over the last two decades. In [10], Korobeinikov studied the global properties of the classical *SEIR* model with the general nonlinear incidence function $f(S,I)$ and established global properties for this rather general case by means of the direct Lyapunov methods. But for some diseases (e.g. tuberculosis, measles), it is reasonable to include an exposed class for those susceptible individuals who are infected with the disease but not yet infectious, after surviving a certain latent period, these individuals pass into the infective class, and then recover into the recovered class.

Time delays are usually used to model the above latent period (see Huang et al. [6], Li and Zou [12] and Abta [16] for fixed latent period and Li et al. [13], Liu et al. [14], Röst and Wu [19] for distributed latency), thus the previous ODE models become to delayed models, which are difficult to handle mathematically. Moreover, due to the variance of latent period among different individuals, it was argued that a more realistic way to incorporate latency into a model is by considering a general distribution function for the length of the latent period (see [13,18,19]).

In [6], Huang et al. considered *SEIR* model with the incidence rate function of type $F(S)\cdot G(I)$ and fixed exposed period. By constructing suitable Lyapunov functionals, the authors proved the global stability of the endemic equilibrium and the disease-free equilibrium for time delays of any length. Motivated by their works, in this paper, we consider the global

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dynamics of the *SEIR* epidemic models with more general incidence rate function $f(S,I)$ and different distribution functions for the length of the latent period.

The paper is organized as follows. In Section 2, the model and preliminaries are presented; in the following section, the main results are stated and proved. In Section 4, we apply the main results into some special cases, which generalizes and improves some previous results in the literatures.

2. The model and the primary results

Our model takes the forms as follows:

$$\begin{cases} \frac{dS(t)}{dt} = \mu - f(S(t),I(t)) - \mu S(t), \\ \frac{dE(t)}{dt} = f(S(t),I(t)) - \sum_{j=1}^{n} \int_0^\infty \alpha_j k_j(\xi) f(S(t-\xi),I(t-\xi))d\xi - \mu E(t) \\ \frac{dI(t)}{dt} = \sum_{j=1}^{n} \int_0^\infty \alpha_j k_j(\xi) f(S(t-\xi),I(t-\xi))d\xi - (\sigma+\mu)I(t), \\ \frac{dR(t)}{dt} = \sigma I(t) - \mu R(t). \end{cases} \tag{1}$$

Here $S(t),\ E(t),\ I(t)$ and $R(t)$ are the class sizes of susceptible, exposed, infective, and temporarily recovered respectively; $f(\cdot,\cdot)$ represents the incidence function (functional response) of the infectious; and $k_i(s)$ is the delay kernels with $\int_0^\infty k_i(\xi)\mathrm{d}\xi = 1$, and so $k_j(\xi)e^{-\mu\xi}$ represents the rate of those susceptible that had got infected at tune $t-\xi$ and became infective at time t. $\alpha_j > 0,\ j = 1,\ldots,n$ are constants with $\sum_{j=1}^{n}\alpha_j = 1$.

Throughout, we assume that the incidence function $f(\cdot,\cdot)$ satisfy the following assumptions:

(H1) f is non-negative differentiable function with $f(S,0) = f(0,I) = 0$ for all $S,I \geqslant 0$ and $f(S,I) > 0$ for all $S,I > 0$.

(H2)

$$\lim_{I\to 0}\frac{f(1,I)}{f(S,I)} - 1 = \begin{cases} >0 & \text{if } S \in [0,1); \\ <0 & \text{if } S \geqslant 1. \end{cases}$$

(H3)

$$f(S,I) \leqslant I \cdot \frac{\partial f(S,0)}{\partial I} \quad \textit{for all } S,I > 0.$$

Remark 1. When there is no delay in model (1), then it reduces to the model studied by Korobeinikov [10]; Model (1) also extends the model (1) in Huang et al. [6] by generalizing the previous incidence function $F(S)\cdot G(S)$ into general $f(S,I)$, and extending the previous fixed latency into distributed latency.

Let $N(t) = S(t) + E(t) + I(t) + R(t)$, thus from (1), we directly have

$$\frac{dN(t)}{dt} = \mu - \mu N(t), \quad t > 0.$$

Thus we have

$$\lim_{t\to\infty} N(t) = 1. \tag{2}$$

Since $E(t),\ R(t)$ are completely determined by $S(t)$ and $I(t)$ in model (1), thus we only need to consider the behaviors of its sub-system:

$$\begin{cases} \frac{dS(t)}{dt} = \mu - f(S(t),I(t)) - \mu S(t), \\ \frac{dI(t)}{dt} = \sum_{j=1}^{n} \int_0^\infty \alpha_j k_j(\xi) f(S(t-\xi),I(t-\xi))d\xi - (\sigma+\mu)I(t). \end{cases} \tag{3}$$

The initial condition for model (3) is

$$S(\theta) = \varphi_1(\theta) \geqslant 0,\ I(0) = \varphi_2(\theta) \geqslant 0,\ \theta \leqslant 0,\ \text{and } \varphi_1(0), \varphi_2(0) > 0.$$

Model (3) contains infinite delays, thus its associated initial conditions will be restricted in an appropriate fading memory space (for the general theory and applications, we refer to [2,5,11]). For any $\Delta \in (0,b)$, we define

$$C_\Delta = \{\varphi : \mathbb{R}_{\leqslant 0} \to \mathbb{R} : \varphi(\theta)e^{\Delta\theta} \text{ is bounded, uniformly continuous and differentiable}\}$$

and

$$Y_\Delta = \{\varphi \in C_\Delta : \varphi(\theta) \geqslant 0 \text{ for all } \theta \leqslant 0\}$$

with the norm on C_Δ and Y_Δ given by $\|\varphi\| = \sup_{\theta\leqslant 0}\|\varphi(\theta)e^{\Delta\theta}\|$. Thus we have $\varphi(0) \leqslant \|\varphi\|$.

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