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Improved methods for the simultaneous inclusion of multiple polynomial zeros ☆

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ABSTRACT

Using a new fixed point relation, the interval methods for the simultaneous inclusion of complex multiple zeros in circular complex arithmetic are constructed. Using the concept of the *R*-order of convergence of mutually dependent sequences, we present the convergence analysis for the total-step and the single-step methods with Schröder's and Halley-like corrections under computationally verifiable initial conditions. The suggested algorithms possess a great computational efficiency since the increase of the convergence rate is attained without additional calculations. Two numerical examples are given to demonstrate convergence characteristics of the proposed method.
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1. Introduction

The goal of this paper is to present algorithms for the simultaneous inclusion of simple or multiple complex zeros of a polynomial. These algorithms are realized in circular complex arithmetic. We recall that the main advantage of circular arithmetic methods lies in automatic computation of rigorous error bound (given by the radii of resulting inclusion disks) on approximate solutions. It turned out that circular interval complex arithmetic is useful, elegant and powerful tool for finding errors in the sought results.

Problems in engineering and many other branches often reduce to the location or determination of polynomial zeros, which points that algorithms for finding polynomial zeros are a natural, powerful, and versatile tool. These algorithms are used for solving problems of different kinds in many disciplines such as differential and difference equations, eigenvalue problems, various mathematical models, theory of control systems [5,11,29], image processing [18], statistical physics [38], theoretical computer science [34], stability of systems [4,15], analysis of transfer functions [12], economics and finance [13, 16, Ch. 9], mathematical biology [6,17], evolutionary game theory [19,20,30–33] and other branches. The listed problems belonging various disciplines led to a rich blend of mathematics, numerical analysis and applied sciences.

A great importance of the problem of approximating polynomial zeros in the theory and practice led to a vast numerical algorithms in this topic, see the books [2,22,27], the papers [7–10,23–26,35–37] and references cited therein.

The presentation of the paper is organized as follows. Some basic definitions and operations of circular complex interval arithmetic, necessary for the convergence analysis and the construction of inclusion methods, are given in Introduction. The basic total-step and single-step methods are derived in Section 2. In Section 3 we develop the improved total-step methods

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with Schröder's and Halley-like corrections. Convergence analysis under computationally verifiable initial conditions is given in Sections 4 and 5. In Section 6 we discuss the single-step versions of improved total step methods, while the numerical examples are presented in Section 7.

The development and convergence analysis of the proposed algorithms need the basic properties of the so-called circular complex arithmetic introduced by Gargantini and Henrici [10]. A circular closed region (disk) $Z := \{z : |z-c| \leqslant r\}$ with center $c := \operatorname{mid} Z$ and radius $r := \operatorname{rad} Z$ is denoted by parametric notation $Z := \{c; r\}$. If $Z_k := \{c_k; r_k\}$ $(k = 1, 2)$, then

$$\begin{aligned} Z_1 \pm Z_2 &= \{c_1 \pm c_2; r_1 + r_2\}, \\ Z_1 \cdot Z_2 &= \{c_1 c_2; |c_1| r_2 + |c_2| r_1 + r_1 r_2\}, \end{aligned} \tag{1}$$

$$Z^{-1} = \{c; r\}^{-1} = \frac{\{\bar{c}; r\}}{|c|^2 - r^2} \quad (|c| > r,\ i.e.\ 0 \notin Z) \quad \text{(exact inversion)}, \tag{2}$$

$$Z^{I_c} = \{c; r\}^{I_c} = \left\{\frac{1}{c}; \frac{r}{|c|(|c|-r)}\right\} \quad (|c| > r,\ i.e.\ 0 \notin Z) \quad \text{(centered inversion)}, \tag{3}$$

$$Z_1 : Z_2 = Z_1 \cdot Z_2^{-1} \quad \text{or} \quad Z_1 : Z_2 = Z_1 \cdot Z_2^{I_c} \quad (0 \notin Z_2).$$

Note that $Z^{-1} \subset Z^{I_c}$, meaning that the exact inversion always produces smaller disks. However, the centered inversion Z^{I_c} has two important advantages: it provides simpler computation and preserves centered form of results, that is, $\operatorname{mid} f(\{c; r\}) = 1/f(c)$.

For the basic interval operations $+, -, \cdot, :$ the *inclusion property* is valid:

$$Z_k \subseteq W_k \Rightarrow Z_1 \ * Z_2 \subseteq W_1 \ * W_2 \quad (k = 1, 2; * \in \{+, -, \cdot, :\}).$$

Moreover, if f is a rational function and F its *complex circular extension*, then

$$Z_k \subseteq W_k \ (k = 1, \ldots, q) \Rightarrow F(Z_1, \ldots, Z_q) \subseteq F(W_1, \ldots, W_q).$$

In particular, we have

$$w_k \in W_k \ (k = 1, \ldots, q;\ w_k \in \mathbb{C}) \Rightarrow f(w_1, \ldots, w_q) \in F(W_1, \ldots, W_q).$$

In this paper we will use the following obvious properties:

$$z \in \{c; r\} \Longleftrightarrow |z - c| \leqslant r, \tag{4}$$

$$\{c_1; r_1\} \cap \{c_2; r_2\} = \emptyset \Longleftrightarrow |c_1 - c_2| > r_1 + r_2. \tag{5}$$

More details about circular arithmetic can be found in the books [2,22,27]. Throughout this paper disks in the complex plane will be denoted by capital letters.

2. Basic methods

Let $P(z) = z^n + a_{n-1} z^{n-1} + \ldots + a_1 z + a_0$ $(n \geq 3)$ be a monic polynomial with simple or multiple complex zeros $\zeta_1, \ldots, \zeta_\nu$ $(2 \leqslant \nu \leqslant n)$ of multiplicities $\mu_1, \ldots, \mu_\nu$ $(\mu_1 + \ldots + \mu_\nu = n)$, respectively, and let $z_1, \ldots, z_\nu$ be their approximations. For the point $z = z_i$ let us introduce the notations:

$$\begin{aligned} \Sigma_{k,i} &= \sum_{j \in \boldsymbol{I}_\nu \setminus \{i\}} \tfrac{\mu_j}{(z_i - \zeta_j)^k}, \quad s_{k,i} = \sum_{j \in \boldsymbol{I}_\nu \setminus \{i\}} \tfrac{\mu_j}{(z_i - z_j)^k} \quad (k = 1, 2), \\ \delta_{1,i} &= \tfrac{P'(z_i)}{P(z_i)}, \quad \delta_{2,i} = \tfrac{P'(z_i)^2 - P(z_i) P''(z_i)}{P(z_i)^2}, \quad u_i = \tfrac{P(z_i)}{P'(z_i)}, \\ \varepsilon_i &= z_i - \zeta_i, \end{aligned} \tag{6}$$

where $\boldsymbol{I}_\nu = \{1, \ldots, \nu\}$ is the iteration index. Assume that we have found disjoint disks $Z_1, \ldots, Z_\nu$ such that each of them contains one and only one zero of P, that is, $\zeta_i \in Z_i$ $(i \in \boldsymbol{I}_\nu)$. Define the inclusion disks

$$S_{k,i} = \sum_{j \in \boldsymbol{I}_\nu \setminus \{i\}} \frac{\mu_j}{(z_i - Z_j)^k} \quad (k = 1, 2;\ i \in \boldsymbol{I}_\nu)$$

and introduce the disks

$$S_{k,i}(\boldsymbol{X}, \boldsymbol{W}) = \sum_{j=1}^{i-1} \left(\mathrm{INV}_1(z - X_j)\right)^k + \sum_{j=i+1}^{\nu} \left(\mathrm{INV}_1(z - W_j)\right)^k \quad (k = 1, 2), \tag{7}$$

$$B_i(\boldsymbol{X}, \boldsymbol{W}) = \mu_i u_i \left(1 - \mu_i + \mu_i u_i \frac{P''(z_i)}{P'(z_i)} - u_i^2 \left(S_{1,i}^2(\boldsymbol{X}, \boldsymbol{W}) - \mu_i S_{2,i}(\boldsymbol{X}, \boldsymbol{W})\right)\right), \tag{8}$$

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