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Persistent fluctuations in a dual model with frictions: The role of delays

Luca Guerrini [a,*], Mauro Sodini [b]

[a] Department of Management, Polytechnic University of Marche, Piazza Martelli 8, 60121 Ancona, Italy
[b] Department of Economics and Management, University of Pisa, via Cosimo Ridolfi 10, 56124 Pisa, Italy

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ABSTRACT

This article studies the dynamics in a simple dual model where agents may shift from the traditional sector to the manufacturing one. We show that a delay between the exit from the traditional sector and the possibility to be employed in the manufacturing one is a source of endogenous fluctuations. In particular, by choosing time delay as the bifurcation parameter, it is proved that the system may lose stability and a supercritical Hopf bifurcation occurs when time delay passes through a critical value.

1. Introduction

Since the pioneeristic pioneering contributions of Lewis [13], many several scholars have used the concept of dual economy in order to investigate some important features of economic systems. A dual economy is a stage in development characterized by the coexistence of two sectors with structural differences in technology, allocation of resources or demand. This kind of models are useful to explain phenomena such as internal migration from the rural sector to the urban sector (see the seminal paper of Harris and Todaro [11]); international mobility of unskilled or skilled workers (see Chaudhuri [4]); or the coexistence and evolution of formal sectors characterized by search frictions, and informal and competitive sectors (see Zenou [17] and Satchi and Temple [15]). In addition, dual economy models focused on the division between agriculture and non-agriculture sectors have proven to be effective in explaining the international distribution of income (see Dietrich [6]), or the coevolution of HIV infection and economic growth in developing countries (see Cuddington [2]).

With regard to the dynamic properties of this kind of models, Matsuyama [14] studies a dual economy described by a Hamiltonian system and, by means of bifurcation techniques, analyses the role of expectations and public policy in the emergence of underdevelopment traps or in the existence of takeoff path, while De Palma and Seegmuller [5] consider a dual labor market and show the possibility of endogenous fluctuations in a discrete time context.

The aim of this paper is to extend the work of Cai [3] focused on job migration. In particular, we allow for the existence of a delay between the exit from the traditional sector and the possibility to be employed in the manufacturing sector, and we show that this mechanism may cause long-term oscillations. It is worth to be stressed that in contrast to a large part of literature on economic models with delays (see e.g. [1,7–10,16]), the existence of fluctuations is related to the intensity of the friction in labour market and it is not induced by lags in capital formation. The economic intuition of this result is that the mismatch between the decisions of leaving the job in the traditional sector and the effective entry in the advanced sector may create mechanisms of feedback generating nonlinear dynamics.

* Corresponding author.
E-mail addresses: luca.guerrini@univpm.it (L. Guerrini), m.sodini@ec.unipi.it (M. Sodini).

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From a mathematical point of view, the dynamics of the model is described by a two-dimensional continuous dynamical system with delay. By applying the techniques of Hassard et al. [12] on the detection of limit cycles, we show the possibility of persistent oscillations. In particular, by choosing the time delay as a bifurcation parameter, it is shown that the system may undergo a supercritical Hopf bifurcation and a stable cycle may emerge.

The rest of the paper proceeds as follows: Section 2 builds on the model. Section 3 characterizes the dynamic properties of the steady state. Section 4 is focused on the analysis of the Hopf bifurcation. Section 5 shows some numerical experiments and Section 6 concludes.

2. The model

We consider an economy with two sectors: the manufacturing sector (denoted by *I*) and the agricultural sector (denoted by *II*). Sector *I* is competitive and capital accumulation of physical capital K is described by the following Solow-like equation:

$$\dot{K} = sF(K, L) - \delta K, \tag{1}$$

where s denotes the saving rate assumed fixed. Wage w is paid at the marginal product of labor that is $w(K, L) = \partial F(K, L)/\partial L$. Sector *II* is noncompetitive and the wage rate is fixed at the survival level w_0. According to the Cai's model [3], we assume that agents may shift from the two sectors. Specifically, such a shift depends on the wage differential between sectors and it is defined as the difference between $w(K, L)$ and w_0 but a congestion effect exists in Sector *I*. The main novelty of this work is the assumption that migration from Sector *II* to Sector *I* does not convert immediately in new labour force in Sector *I*, but a gestation lag exists, due to possible difficulties to find a vacancy or to acquire necessary skills for job in the manufacturing sector.

Let L_0 be the initial labor supply in sector *I*, and M the total labor shifted from Sector *II* to Sector *I*. Thus, the aggregate labor in sector *I* is $L_0 + M$.

The shift rate is described by

$$\dot{M} = G(w(K, L_0 + M_d) - w_0) - H(M_d), \tag{2}$$

where $M(t-\tau) = M_d$, $w(K, L_0 + M_d)$ is the wage rate of sector *I* and w_0 is the survival wage rate of Sector *II*. We assume that G and H are C^3 functions satisfying $G(0) = H(0) = 0, G'(\cdot) > 0, H'(\cdot) > 0, H(\infty) = \infty$.

Remark 1. The key feature of equation (2) is that the migration flow is governed by the wage differential formed on the basis of migrants at time $t-\tau$ and not on the current state of M.

From (1) and (2), one can obtain the following new system of non-linear delay differential equations

$$\dot{K} = sF(K, L_0 + M_d) - \delta K, \tag{3}$$

$$\dot{M} = G(w(K, L_0 + M_d) - w_0) - H(M_d). \tag{4}$$

It is clear that critical points of system (3) and (4) correspond to those with vanishing delay. From Cai [2], we know there exists a unique non-trivial equilibrium (K_*, M_*), where $sF(K_*, L_0 + M_*) = \delta K_*$ and $G(w(K_*, L_0 + M_*) - w_0) = H(M_*)$.

3. Stability and Hopf bifurcations

In this section, we study the stability of the positive equilibrium and the existence of local Hopf bifurcations. Let $x = K - K_*$ and $y = M - M_*$. Then (3) and (4) can be transformed into the following form

$$\dot{x} = sF(x + K_*, L_0 + y_d + M_*) - \delta(x + K_*), \tag{5}$$

$$\dot{y} = G(w(x + K_*, L_0 + y_d + M_*) - w_0) - H(y_d + M_*). \tag{6}$$

The linearization of (5) and (6) at (0,0) is

$$\dot{x} = ax + by_d, \tag{7}$$

$$\dot{y} = cx + dy_d, \tag{8}$$

where

$$a = sF_K(K_*, L_0 + M_*) - \delta = \frac{s}{K_*}[K_* F_K(K_*, L_0 + M_*) - F(K_*, L_0 + M_*)] < 0,$$
$$b = sF_{M_d}(K_*, L_0 + M_*) > 0,$$
$$c = G_K(w(K_*, L_0 + M_*) - w_0)F_{M_d K}(K_*, L_0 + M_*) > 0,$$
$$d = G_{M_d}(w(K_*, L_0 + M_*) - w_0)F_{M_d M_d}(K_*, L_0 + M_*) - H_{M_d}(M_*) < 0.$$