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The influence of wall flexibility on unsteady peristaltic flow of Prandtl fluid in a three dimensional rectangular duct

Arshad Riaz [a,d,*], S. Nadeem [b], R. Ellahi [a], Noreen Sher Akbar [c]

[a] Department of Mathematics and Statistics, FBAS, IIU, Islamabad 44000, Pakistan
[b] Department of Mathematics, Quaid-i-Azam University 45320, Islamabad 44000, Pakistan
[c] DBS&H, CEME, National University of Sciences and Technology, Islamabad, Pakistan
[d] Department of Mathematics, UOS, Lahore Campus, Pakistan

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ABSTRACT

The influence of walls attributes on the peristaltic transport in a three dimensional rectangular channel has been incorporated in this article. It is found that the idea of three dimensional rectangular channel is quite helpful as compared with the two dimensional channels/tubes. The flow is considered to be incompressible and unsteady. Cartesian coordinate system has been employed to carry out the mathematical investigation. The obtained highly non-linear and non-homogeneous partial differential equations have been solved analytically by using homotopy perturbation method. The discussion on possible physical aspects of all emerging parameters appeared in the problems has been revealed through set of graphs sketched for velocity and stream function.

1. Introduction

Peristaltic pumping is a form of fluid transport that occurs when a progressive wave of area contraction or expansion propagates along the length of a distensible tube containing the fluid. In many physiological situations, peristalsis is used by the body to propel or mix the contents of a tube, for example, in ureter, gastro-intestinal tract, the bile duct and the other glandular ducts. The need for peristaltic pumping also arises in circumstances where it is desirable to avoid using any internal moving parts such as pistons, in pumping process [1]. The research on peristaltic flows has achieved the contemplation of many researchers after the pioneering work of Latham [2]. Most of the biofluids are non-Newtonian in nature and do not follow the linear relationship between stress and deformation. The concerned studies can be cited in Refs. [3–6]. Peristaltic flows of Newtonian and non-Newtonian fluids has been presented by many researchers [7–10].

In the present time, many of the researchers have contended to investigate the peristaltic phenomenon in compliant (flexible) walls channels/tubes. Almost all vessels carrying fluids within the body are flexible, and interactions between an internal flow and wall deformation often underlie a vessel's biological function or dysfunction. Such interactions can involve a rich range of fluid-mechanical phenomena, including nonlinear pressure-drop/flow-rate relations, self-excited oscillations of single-phase flow at high Reynolds number and capillary-elastic instabilities of two-phase flow at low Reynolds number [11]. Mitra and Prasad [12] have worked on the influence of wall properties and Poiseuille flow in peristalsis. A new model for study the effect of wall properties on peristaltic transport of a viscous fluid has been investigated by Elnaby and Haroun [13]. More studies on the peristaltic flow of Newtonian and non-Newtonian fluids have been made in

* Corresponding author at: Department of Mathematics and Statistics, FBAS, IIU, Islamabad 44000, Pakistan.
E-mail address: ariiui@hotmail.com (A. Riaz).

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Refs. [14–16]. The sudy of peristaltic flows in three dimensional geometries has been considered by some of the researchers [17–20]. According to Reddy et. al. [17], the sagittal cross section of the uterus may be better approximated by a tube of rectangular cross section than a two dimensional flow. However, the peristaltic flow of non-Newtonian Prandtl model has not been yet considered by anyone in three dimensional channel having flexible walls.

The motivation of the present work comes from the above discussion and the authors have urgency to work on unsteady peristaltic flow of Prandtl fluid in a compliant rectangular duct having uniform area of cross section. The governing equations have been derived under the assumptions of long wave length and low Reynolds number. The obtained highly nonlinear equations have been solve analytically and the possible physical features of some pertinent parameters have been discussed through a series of graphs for velocity profile and stream functions.

2. Mathematical formulation

We consider the peristaltic flow of non-Newtonian Prandtl fluid in a cross section of rectangular channel containing the width $2d$ and height $2a$. The geometry is arranged in Cartesian coordinate system in taken in such a way that $x-$axis is taken along the axial direction, $y-$axis is taken along the lateral direction and $z-$axis is along the normal face of rectangular channel (See Fig. 1). The channel is supposed to having compliant walls. The flow is developed by the peristaltic sinusoidal waves propagating along the walls of the channel.

The walls of the channel are defined as

$$z = \pm\eta(x,t) = \pm a \pm b\cos\left[\frac{2\pi}{\lambda}(x - ct)\right],$$

where a and b are the amplitudes of the waves, λ is the wavelength, c is the velocity of propagation, t is the time and x is the direction of wave propagation. The walls parallel to $xz-$plane remain stable and do not cause any peristaltic wave motion. The stress tensor for the Prandtl fluid model is defined by [14]

$$\mathbf{S} = \frac{A\sin^{-1}\left(\frac{1}{C}\left(\left(\frac{\partial u}{\partial y}\right)^2 + \left(\frac{\partial w}{\partial y}\right)^2\right)^{1/2}\right)}{\left(\left(\frac{\partial u}{\partial y}\right)^2 + \left(\frac{\partial w}{\partial y}\right)^2\right)^{1/2}}\frac{\partial u}{\partial y}, \tag{1}$$

in which A and C represent material constants of Prandtl fluid model. Let $(u, 0, w)$ be the velocity field for a rectangular duct. The observing equations for the flow problem are stated as

Mass conservation equation

$$\frac{\partial u}{\partial x} + \frac{\partial w}{\partial z} = 0, \tag{2}$$

Momentum conservation equation

$$\rho\left(\frac{\partial u}{\partial t} + u\frac{\partial u}{\partial x} + w\frac{\partial u}{\partial z}\right) = -\frac{\partial p}{\partial x} + \frac{\partial}{\partial x}S_{xx} + \frac{\partial}{\partial y}S_{xy} + \frac{\partial}{\partial z}S_{xz}, \tag{3}$$

$$0 = -\frac{\partial p}{\partial y} + \frac{\partial}{\partial x}S_{yx} + \frac{\partial}{\partial y}S_{yy} + \frac{\partial}{\partial zz}S_{yz}, \tag{4}$$

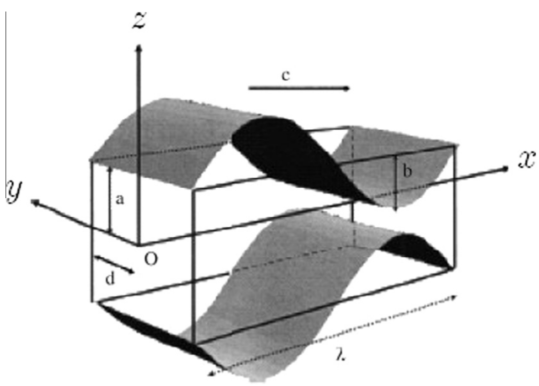

Fig. 1. Schematic diagram for peristaltic flow in a rectangular duct.