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Comments and an improved result on "stability analysis for continuous system with additive time-varying delays: A less conservative result"

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ABSTRACT

This paper points out a technical problem in the theorem and proof in the above paper. Furthermore, by employing a newly-proposed integral inequality, an improved stability criterion for a continuous linear system with two additive time-varying delays is presented. A numerical example is given to show the effectiveness of the proposed result.

1. Introduction

The past several decades have witnessed considerable interests in research on analysis and synthesis of time delay systems due to the inherent existence of delays in practical systems. Recently, recurring attention has been paid to stability analysis for systems with additive time-varying delays arising from the so-called networked control systems (NCSs), see, e.g., [1–5]. In NCSs, the control loops are closed via the communication network and the system components, such as sensors, actuators, controllers are usually physically distributed over the network. As a consequence, the data transmission through the sensor-to-controller channel and the controller-to-actuator channel may be affected by networked-induced delays caused by limited network bandwidth and/or congested network traffic. It should be mentioned that delays induced through different communication channels are generally time-varying and expose different physical characteristics. In this sense, it is not reasonable to lump these two delays together and treat them as one delay, which gives rise to the study on stability analysis for systems with additive time-varying delays.

In this paper, we will point out and correct the technique problem in [1]. Moreover, an improved stability criterion will be derived by using a newly-proposed integral inequality presented in [6]. A numerical example will be given to illustrate the effectiveness of the proposed result.

2. Correction

Consider a continuous linear system with two additive time-varying delays described by

$$\dot{x}(t) = Ax(t) + A_d x(t - \tau_1(t) - \tau_2(t)),\ x(t) = \phi(t),\ t \in [-\bar{\tau}, 0], \tag{1}$$

where $x(t) \in \mathbb{R}^n$ is the state; $\phi(t)$ is the initial condition on the segment $[-\bar{\tau}, 0]$; A and A_d are known system matrices of appropriate dimensions; $\tau_1(t)$ and $\tau_2(t)$ represents two delay components in the state with different physical characteristics satisfying

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$$0 \leqslant \tau_1(t) \leqslant \bar{\tau}_1 < \infty,\ \dot{\tau}_1(t) \leqslant d_1 < \infty;\ 0 \leqslant \tau_2(t) \leqslant \bar{\tau}_2 < \infty,\ \dot{\tau}_1(t) \leqslant d_2 < \infty. \tag{2}$$

To facilitate the discussion, we denote $\tau(t) = \tau_1(t) + \tau_2(t)$, $\bar{\tau} = \bar{\tau}_1 + \bar{\tau}_2$, and $d = d_1 + d_2$.

In order to obtain a delay-dependent stability criterion for the system (1), the following Lyapunov–Krasovskii functional candidate is chosen in [1]

$$V(t) = V_1(t) + V_2(t) + V_3(t), \tag{3}$$

where

$$V_1(t) = x^T(s)Px(t), \tag{4}$$

$$V_2(t) = \int_{t-\tau(t)}^{t} x^T(s)Q_1x(s)ds + \int_{t-\tau_1(t)}^{t} x^T(s)Q_2x(s)ds + \int_{t-\tau(t)}^{t-\tau_1(t)} x^T(s)Q_3x(s)ds, \tag{5}$$

$$V_3(t) = \int_{t-\bar{\tau}}^{t}\int_{\theta}^{t} \dot{x}^T(s)R_1\dot{x}(s)dsd\theta + \int_{t-\bar{\tau}_1}^{t}\int_{\theta}^{t} \dot{x}^T(s)R_2\dot{x}(s)dsd\theta + \int_{t-\bar{\tau}}^{t-\bar{\tau}_1}\int_{\theta}^{t} \dot{x}^T(s)R_3\dot{x}(s)dsd\theta. \tag{6}$$

In the proof of Theorem 1 in [1], the first step is to take the time derivative of the Lyapunov–Krasovskii functional along the trajectory of the system (1), which yields

$$\dot{V}_1(t) = 2x^T(s)P\dot{x}(t), \tag{7}$$

$$\dot{V}_2(t) \leqslant x^T(t)(Q_1 + Q_2)x(t) - (1-d_1)x^T(t-\tau_1(t))(Q_2 - Q_3)x(t-\tau_1(t)) - (1-d)x^T(t-\tau(t))(Q_1 + Q_3)x(t-\tau(t)), \tag{8}$$

$$\dot{V}_3(t) = \dot{x}^T(t)(\bar{\tau}R_1 + \bar{\tau}_1R_2 + \bar{\tau}_2R_3)\dot{x}(t) - \int_{t-\bar{\tau}}^{t} \dot{x}^T(s)R_1\dot{x}(s)ds - \int_{t-\bar{\tau}_1}^{t} \dot{x}^T(s)R_2\dot{x}(s)ds - \int_{t-\bar{\tau}}^{t-\bar{\tau}_1} \dot{x}^T(s)R_3\dot{x}(s)ds, \tag{9}$$

with the constraint $Q_2 \geqslant Q_3 \geqslant 0$.

Next, to estimate the upper bound of the derivative of the Lyapunov–Krasovskii functional $V(t)$, the authors in [1] apply the following bounding inequalities to bound the last three terms of $\dot{V}_3(t)$

$$-\int_{t-\bar{\tau}}^{t} \dot{x}^T(s)R_1\dot{x}(s)ds \leqslant -\int_{t-\tau(t)}^{t} \dot{x}^T(s)R_1\dot{x}(s)ds, \tag{10}$$

$$-\int_{t-\bar{\tau}_1}^{t} \dot{x}^T(s)R_2\dot{x}(s)ds \leqslant -\int_{t-\tau_1(t)}^{t} \dot{x}^T(s)R_2\dot{x}(s)ds, \tag{11}$$

$$-\int_{t-\bar{\tau}}^{t-\bar{\tau}_1} \dot{x}^T(s)R_3\dot{x}(s)ds \leqslant -\int_{t-\tau(t)}^{t-\tau_1(t)} \dot{x}^T(s)R_3\dot{x}(s)ds. \tag{12}$$

However, it should be pointed out that there is no guarantee that the inequality (12) holds. In fact, the left hand side of the inequality (12) can be rewritten as

$$\begin{aligned} -\int_{t-\bar{\tau}}^{t-\bar{\tau}_1} \dot{x}^T(s)R_3\dot{x}(s)ds &= -\int_{t-\tau(t)}^{t-\bar{\tau}_1} \dot{x}^T(s)R_3\dot{x}(s)ds - \int_{t-\bar{\tau}}^{t-\tau(t)} \dot{x}^T(s)R_3\dot{x}(s)ds \\ &= -\int_{t-\tau(t)}^{t-\tau_1(t)} \dot{x}^T(s)R_3\dot{x}(s)ds - \int_{t-\tau_1(t)}^{t-\bar{\tau}_1} \dot{x}^T(s)R_3\dot{x}(s)ds - \int_{t-\bar{\tau}}^{t-\tau(t)} \dot{x}^T(s)R_3\dot{x}(s)ds. \end{aligned} \tag{13}$$

Obviously, $-\int_{t-\tau_1(t)}^{t-\bar{\tau}_1} \dot{x}^T(s)R_3\dot{x}(s)ds \geqslant 0$ and $-\int_{t-\bar{\tau}}^{t-\tau(t)}\dot{x}^T(s)R_3\dot{x}(s)ds \leq 0$. As a result, the inequality (12) may not be satisfied when the sum of those two terms is greater than zero. Therefore, the direct use of the inequality (12) to estimate the upper bound of the derivative of the Lyapunov–Krasovskii functional $V(t)$ inevitably leads to a misleading statement in the Theorem 1 in [1].

We now present the corrected result by exposing some constraint on the Lyapunov matrices R_2 and R_3. Understanding that

$$-\int_{t-\bar{\tau}_1}^{t} \dot{x}^T(s)R_2\dot{x}(s)ds = -\int_{t-\tau_1(t)}^{t} \dot{x}^T(s)R_2\dot{x}(s)ds - \int_{t-\bar{\tau}_1}^{t-\tau_1(t)} \dot{x}^T(s)R_2\dot{x}(s)ds \tag{14}$$

and combining (13), one obtains