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Cyclic pursuit problems in the two dimensional sphere

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ABSTRACT

In this paper we consider several generalized pursuit models in order to investigate computationally and analytically the asymptotic behavior of a system of n particles moving in the two dimensional sphere. We will consider two scenarios of pursuit, the first one given by the Limit cyclic pursuit with subcases given by using variable or constant speed. This scenario corresponds to a system of particles where one of them follows a trajectory given by a fixed curve and it is followed cyclically by a system of $n-1$ particles using the shortest path between them. The second one is considering a set of particles moving on a sphere under a symmetric or non symmetric cyclic pursuit mechanism. The analysis will be presented with several simulations.

1. Introduction

Pursuit problems have been recently received extensively scientific interest, see for example [1–4]. The basic setup involves two individuals, one of them been pursued by the other individual with its line of sight pointing towards the target. This basic pursuit problem consists in determining the trajectory of effective capture. Pierre Bouguer in 1732 was among the first to use a modern approach of pursuit using mathematical analysis. Since then, a lot of research has been done in several areas of mathematics and computer sciences [5–7].

There has been many generalizations of the pursuit problem with a lot of mechanism and strategies involved. One of the most famous is the pursuit problem with the pursed moving in a circumference with its origins have been traced back to 1748 with a pursuit problem involving a spider chasing a fly around the edge of a semicircular pane of glass. But the most modern known version was proposed by Hathaway in 1921, [8]. For a nice review of the history of pursuit problems, see [9,10]. Among the generalizations of the pursuit problem, the cyclic pursuit, also known as the n-bug problem, is one of the most important. A famous example is the n bug problem, where n bugs, initially located at the vertices of a n-regular polygon, chase each other cyclically using pure pursuit and the problem is to determine all the pursuit curves. Recently, several models were proposed to study geometric asymptotic properties of systems of individuals under various types of pursuit [2]. The goal was to classify them according to their asymptotic behavior such as fixed points, limit cycles and periodic behavior. Two cases were analyzed in depth. In the first one, n particles were considered whose motion is restricted to the plane with the following mechanism of persecution: Particle v_j (for $j=1,\dots,n-1$) follows v_{j+1}, and the particle v_n follows t trajectory chosen given by the orbit of a van der Pol oscillator, which in the linear case reduces to a unit circle with angular frequency ω_n. The individuals (for $j=1,\dots,n-1$) for the reduction case tend asymptotically to circular paths centered at the origin with radius (see Fig. 1).

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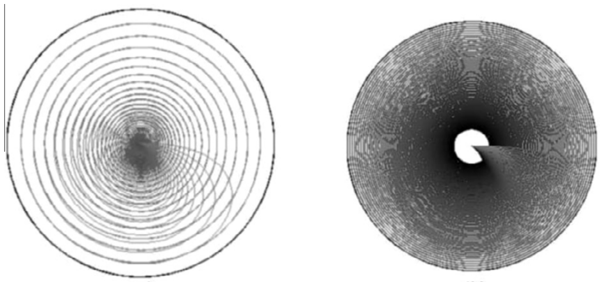

Fig. 1. Case one. $n = 200$ individuals, (a) $\omega_n = 0.8$ (b) $\omega_n = 0.2$.

$$r_j = \frac{1}{(1+\omega_n^2)^{(n-j)/2}}, \quad j = 1, 2, \ldots, n-1. \tag{1}$$

In the second case: a configuration involving n ordered individuals or particles $v_1, v_2, \ldots, v_n$. At any moment, the particle $v_j, j = 1, 2, \ldots, n$, moves directly toward the particle v_{j+r}, for a fixed value of r. The index is assessed module n. There are several cases of asymptotic behavior, one of them characterized by periodic behavior and the rest of them by different attractors, namely equilibria which correspond to the mutual pursuit of individuals and finally a limit cycle corresponding to the case of pursuit of an independent individual. For $r = 1$ there is only one attractor point given by arithmetic mean of the initial conditions. More precisely the global attractor for a 1-cyclic system is characterized by the barycenter of the initial conditions at each stage. In Fig. 2 we show the $n = 20$ individuals case in the plane with random initial conditions.

Our goal in this work is to consider several generalized pursuit models when the movement of a system of n particles is restricted to the two dimensional sphere, although the results generalize easily to higher dimensional spheres. We will consider two cases of pursuit. The first one is what we call the Limit cyclic pursuit with subcases given by using variable or constant speed. This case corresponds to a system of particles where one of them follows a trajectory given by a fixed curve and it is followed cyclically by a system of $n - 1$ particles using the shortest path between them at every time. The second one is considering a set of particles moving on a sphere under a symmetric or non symmetric cyclic pursuit mechanism.

It is important to remark that there are other studies of the symmetrical cyclic pursuit problem in the sphere, in the hyperbolic plane and in other surfaces [11], although in a different manner than we shall do here.

This paper is organized as follows. In Section 2 we study the geodesic trajectories in the sphere. In Section 3 we introduce the pursuit dynamics in the sphere, namely the Limit cyclic pursuit with sub cases given by using variable or constant speed. We analyze the resultant asymptotic behavior and include some numerical simulations in the analysis. Finally the classic cyclic case is analyzed in Section 4.

2. Pursuit trajectories in the sphere

Before we propose the particular pursuit dynamics in the sphere, we need to find in this section the equations governing the movement of a pursuing particle chasing a moving particle with a fixed trajectory in the sphere. The strategy of the pursuing particle is that at any given time it follows a trajectory (a geodesic) with the minimal distance to the chased particle.

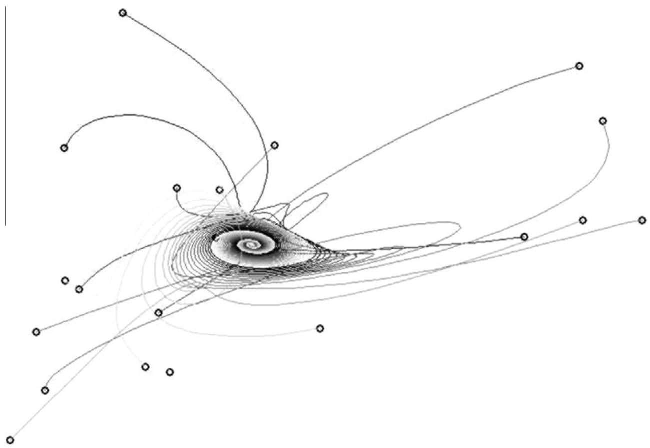

Fig. 2. 1-cyclic pursuit in $\mathbb{R}^2, n = 20$ individuals.