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Some array type polynomials associated with special numbers and polynomials

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ABSTRACT

The main objective in this paper is first to establish new identities for the λ-Stirling type numbers of the second kind, the λ-array type polynomials, the Apostol–Bernoulli polynomials and the Apostol–Bernoulli numbers. We then construct a λ-delta operator and investigate various generating functions for the λ-Bell type numbers and for some new polynomials associated with the λ-array type polynomials. We also derive several other identities and relations for these polynomials and numbers.

1. Introduction, notations and preliminaries

1.1. Notations and motivation

Throughout this paper, we use the following standard notations:

$$\mathbb{N} = \{1, 2, 3, \ldots\}, \quad \mathbb{N}_0 = \{0, 1, 2, 3, \ldots\} = \mathbb{N} \cup \{0\} \quad \text{and} \quad \mathbb{Z}^- = \{-1, -2, -3, \ldots\}.$$

As usual, $\mathbb{Z}$ denotes the set of integers, $\mathbb{R}$ is the set of real numbers, $\mathbb{C}$ is the set of complex numbers and $\mathbb{Z}_p$ is the set of p-adic integers. The principal value $\ln z$ is the logarithm whose imaginary part lies in the interval $(-\pi, \pi]$. Furthermore, in this paper, we use the following notational conventions:

$$0^n = \begin{cases} 1 & (n = 0) \\ 0 & (n \in \mathbb{N}), \end{cases}$$

$$\binom{\lambda}{0} = 1 \quad \text{and} \quad \binom{\lambda}{n} = \frac{\lambda(\lambda-1)\cdots(\lambda-n+1)}{n!} \qquad (n \in \mathbb{N};\ \lambda \in \mathbb{C})$$

and

$$\{\lambda\}_0 = 1 \quad \text{and} \quad \{\lambda\}_n = \prod_{j=0}^{n-1} (\lambda - j) \quad (n \in \mathbb{N};\ \lambda \in \mathbb{C}).$$

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Recently, Chang and Ha [5] studied some properties of the array polynomials $S_k^n(x)$ and $A_k^n(x)$; we recall their definition in Sections 1.2 and 2. Simsek [21] constructed generating functions for generalized Stirling type numbers, the array type polynomials and the Eulerian type polynomials. He also gave many applications and identities related to these polynomials. On the other hand, Cangul et al. [2] studied the generating functions (in terms of array type polynomials) related to the "*presentation theme*". By considering the generating pictures in two different groups and the associated monoid extensions, these and other authors have investigated the related generating functions over the presentations of them. If a monoid presentation satisfies efficiency or inefficiency (while it is minimal), then it always contains a minimal number of generators. Working with a minimal number of elements gives a great opportunity to define related generating functions over this presentation (see, for details, [2–4]).

The Apostol–Bernoulli polynomials $\mathfrak{B}_n^{(\alpha)}(x;\lambda)$ of (real or complex) order α, which were introduced and investigated by Luo and Srivastava [12] (see also [13,14]), are defined by means of the following generating function:

$$\left(\frac{t}{\lambda e^t-1}\right)^{\alpha} e^{xt}=\sum_{n=0}^{\infty}\mathfrak{B}_n^{(\alpha)}(x;\lambda)\,\frac{t^n}{n!} \tag{1}$$

$$(|t|<2\pi \quad \text{when} \quad \lambda=1;\ |t|<\ln\lambda \quad \text{when} \quad \lambda\neq 1),$$

where the order α is *tacitly* restricted to nonnegative integer values whenever $\lambda\neq 1$. These higher-order Apostol–Bernoulli polynomials $\mathfrak{B}_n^{(\alpha)}(x;\lambda)$ have been investigated by many authors (see, for example, [11,16,22,23]; see also [24, Chapter 1]). For the Apostol–Bernoulli polynomials $\mathfrak{B}_n(x;\lambda)$ and the higher-order Apostol–Bernoulli numbers $\mathfrak{B}_n^{(\alpha)}(\lambda)$, we have

$$\mathfrak{B}_n(x;\lambda)=\mathfrak{B}_n^{(1)}(x;\lambda) \quad \text{and} \quad \mathfrak{B}_n^{(\alpha)}(\lambda)=\mathfrak{B}_n^{(\alpha)}(0;\lambda).$$

Moreover, the Bernoulli polynomials $B_n^{(\alpha)}(x)$ of (real or complex) order α and the higher-order Bernoulli numbers $B_n^{(\alpha)}$ are given by

$$B_n^{(\alpha)}(x)=\mathfrak{B}_n^{(\alpha)}(x;1) \quad \text{and} \quad B_n^{(\alpha)}=\mathfrak{B}_n^{(\alpha)}(0;1)=\mathfrak{B}_n^{(\alpha)}(1),$$

so that, for the Bernoulli polynomials $B_n(x)$ and the Bernoulli numbers B_n, we have

$$B_n(x)=B_n^{(1)}(x)=\mathfrak{B}_n^{(1)}(x;1) \quad \text{and} \quad B_n=B_n^{(1)}=\mathfrak{B}_n^{(1)}(0;1)=\mathfrak{B}_n^{(1)}(1).$$

In our present investigation, we choose to consider the case of the Apostol–Bernoulli polynomials $\mathfrak{B}_n^{(\alpha)}(x;\lambda)$ when the order $\alpha=v\ (v\in\mathbb{N}_0)$.

The λ-Stirling numbers of the second kind are defined, in [14,22], as follows:

Definition 1. Let $\lambda\in\mathbb{C}$ and $v\in\mathbb{N}_0$. The generalized λ-Stirling numbers $S(n,v;\lambda)$ of the second kind are defined by means of the following generating function:

$$f_v(t;\lambda):=\frac{(\lambda e^t-1)^v}{v!}=\sum_{n=0}^{\infty}S(n,v;\lambda)\,\frac{t^n}{n!} \tag{2}$$

One can easily see from Definition 1 that

$$S(n,v;1)=S(n,v),$$

where $S(n,k)$ denotes the Stirling numbers of the second kind given by (see [6] and [24, p. 78 *et seq.*])

$$z^n=\sum_{k=0}^{n}S(n,k)\{z\}_k \quad \text{or} \quad \frac{(e^z-1)^k}{k!}=\sum_{n=k}^{\infty}S(n,k)\,\frac{z^n}{n!}.$$

1.2. Generalized array type polynomials

In his recent works, Simsek (see [19,21]) constructed generating functions for the λ-*array type polynomials* related to non-negative real parameters. He derived some elementary properties (including recurrence relations) of these polynomials. Here, in this sequel to Simsek's work, we introduce some generalization and unification of these array type polynomials.

Definition 2. Let a and b be positive real numbers with $a\geqq 1, x\in\mathbb{R}, \lambda\in\mathbb{C}$ and $v\in\mathbb{N}_0$. The generalized array type polynomials $S_v^n(x;a,b;\lambda)$ are defined by the following formula:

$$S_v^n(x;a,b;\lambda)=\frac{1}{v!}\sum_{j=0}^{v}(-1)^{v-j}\binom{v}{j}\lambda^j\left[\ln\left(a^{v-j}b^{x+j}\right)\right]^n. \tag{3}$$

Equivalently, the generalized array type polynomials $S_v^n(x;a,b;\lambda)$ in Definition 2 can be defined, as in [21], by means of the following generating function: