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Continuous-time average consensus under dynamically changing topologies and multiple time-varying delays

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ABSTRACT

The average consensus in continuous-time multi-agent systems with dynamically changing topologies and multiple time-varying delays is studied in this paper. The network topology is captured by weighted digraphs which are weakly connected and balanced. Some feasible linear matrix inequalities are established to determine the allowable upper bounds of multiple delays that guarantee the average consensus of the system. Numerical examples are provided to show the usefulness of the theoretical results.

1. Introduction

During the last decade, cooperative distributed control has received tremendous attention due to its wide applications ranging from cooperative control of unmanned aerial vehicles [1,2], distributed sensor fusion [3,4], flocking and swarming [5–7] to attitude alignment of clusters of satellites [8], etc. In a distributed system consisting of multiple agents, a critical problem is to design appropriate protocols and algorithms such that the group of agents can reach an agreement on a certain quantity of interest in the presence of dynamically changing interaction topologies as well as communication delays. Such agreement or consensus problems include many network coordination problems such as formation of patterns, distributed optimization, self-organization and parallel processing.

In a number of applications concerning multi-agent systems, transmission time delays on the transfer of data between agents may frequently occur because of asymmetric interactions, communication congestion, agents moving or finite transmission speed. Generally, introducing a delay leads to performance reduction or instability [9]. The effect of communication time delays on agents' consensus behavior is first studied in [10] for a continuous-time multi-agent system with switching topology and a common constant communication delay. Sufficient and necessary conditions are established to guarantee consensus under varied assumptions on directed or undirected network information flow, and presence or absence of time delays. In [11], drawing on the stability theory of stochastic differential delay equations, sufficient conditions are given for the leader–follower synchronization of coupled second-order harmonic oscillators with constant delays. For systems with non-uniform constant communication delay, stability criteria are given in [12] by using Lyapunov–Krasovskii approaches. Based on linear matrix inequality (LMI) techniques, the authors of [13,14] analyze average consensus problem for multi-agent networks with time-dependent delays. A unified framework for addressing different types of feedback delays in linear undirected multi-agent systems, e.g. delays affecting only the output of the agents' neighbors, or delays affecting both the agents' own and their neighbors' output, is introduced by Münz et al. [15,16]. By deriving frequency-dependent and

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delay-dependent convex sets containing the eigenvalues of feedback matrices, they manage to obtain set-valued consensus conditions for systems with heterogeneous delays. For a multi-agent system with uniformly quasi-strongly connected switching topology, Münz et al. [17] show the delay robustness in the sense that asymptotic consensus can be achieved for arbitrarily large constant, or time-varying delays once consensus is achieved without delays.

To the best of our knowledge, only a few works in the literature address consensus problems in the case of multiple time-varying delay. The theoretical analysis is more challenging in this case. Using LMI method, continuous-time average consensus protocol is proposed in [18] for undirected networks of dynamic agents with multiple time-varying delays. A special matrix is introduced to overcome the singularity of system matrix. The authors of [19,20] transform the original singular close-loop system into a reduced dimensional stable system by employing orthogonal transformations. The maximal allowed upper bounds of multiple time-varying delays are expressed in terms of several LMIs. A common condition in these works is that they assume undirected network topology. One aim of this paper is to extend these results to balanced digraphs (directed graphs), which provide needed flexibility for practical applications. We mention that average consensus problem and leader-following consensus problem for high-order integrator agents with multiple time-varying delay are also dealt with in [21,22], respectively.

The above works [18–22] implicitly assume that the interaction strengths, i.e., link weights, are either unchanged (in the case of fixed topology) or ergodically selected among a finite collection of possibilities (in the case of switching topology). Nevertheless, interaction topologies in most real-world complex networks where coordinated control is expected to have an impact are dynamically changing. In wireless sensor networks and mobile ad hoc networks, for example, existing links may fail due to interference (such as draining of batteries and appearance of an obstacle), and new links may emerge because two sensors come to an effective range of detection with respect to each other. In wired communication networks, messages and packets may be dropped due to buffer overflow on a dynamical basis. Consensus problems of discrete/continuous-time multi-agent systems under dynamically changing topologies are already studied in some papers (see e.g. [23–29]). However, little has been known about average consensus problem along this line of research. Notice that average consensus is a more demanding task since an extra condition on the final value is imposed.

Motivated by the above considerations, in this paper we address average consensus problems in continuous-time multi-agent systems under dynamically changing interaction topologies and multiple time-varying delays. The underlying network is captured as a weighted directed graph. By means of LMI method, we prove that the group of dynamic agents can achieve average consensus asymptotically for appropriate multiple time-varying delays if the network topology is weakly connected and balanced. Note that addressing average consensus is a more challenging task than simply proving the existence of a common limiting state as is done in the general delay robustness issues (see e.g. [15–17]). The maximal allowable upper bounds of multiple time-varying delays can be derived from feasible LMIs by employing Matlab's LMI Toolbox [30]. Moreover, the rate of convergence is estimated. Numerical examples are provided to demonstrate the availability of our theoretical results.

The rest of the paper is organized as follows. In Section 2, we describe our model, and provide the notation to be used throughout the paper, along with some background on algebraic graph theory. In Section 3, we present average consensus analysis for the system. Section 4 contains some simulation results. Finally, Section 5 summarizes the paper and draws directions for future research.

2. Problem statement

In multi-agent systems, the exchange of information between agents can be conveniently captured by a weighted digraph $G = (V, E, A)$ of order n, where $V = \{1, 2, \dots, n\}$ is the set of nodes, $E \subseteq V \times V$ is the set of edges, and $A = (a_{ij})$ is the weighted adjacency matrix. A directed edge from node j to node i, denoted by (j, i), exists if and only if $a_{ij} > 0$. We assume that $a_{ii} = 0$ for all $i \in V$. The set of neighbors of node i is denoted by $N_i = \{j \in V : (j, i) \in E\}$. The graph Laplacian, i.e., Laplacian matrix, $L = (l_{ij})$ induced by the digraph G is defined by [31]

$$l_{ij} = \begin{cases} -a_{ij}, & i \neq j, \\ \sum_{k=1}^{n} a_{ik}, & i = j. \end{cases} \tag{1}$$

A digraph G is said to be balanced [10] if $\sum_{j=1}^{n} a_{ij} = \sum_{j=1}^{n} a_{ji}$ for all $i \in V$. Clearly, a digraph is balanced if and only if the total weight of edges entering a vertex and leaving the same vertex are equal for all vertices. By definition, an undirected graph is balanced. An important property of balanced digraphs is that $\mathbf{1} = (1, \dots, 1)^T \in \mathbb{R}^n$ is a left eigenvector of their Laplacian, i.e., $\mathbf{1}^T L = 0$. Here, T represents the transpose operation.

Consider a continuous-time system consisting of n agents, which is depicted by a digraph G, with dynamics given by

$$\dot{x}_i(t) = u_i(t) \tag{2}$$

for $i = 1, \dots, n$, where $x_i(t) \in \mathbb{R}$ is the state (or value) of agent i at time t, and $u_i(t) \in \mathbb{R}$ is the control input at time t. Denote by $x = (x_1, \dots, x_n)^T$. We say system (2) achieves average consensus asymptotically if for any initial value $x(0) \in \mathbb{R}^n$, the states of agents satisfy $x_i(t) \to \mathrm{Ave}(x(0)) := \left(\sum_{i=1}^{n} x_i(0)\right)/n$ as $t \to \infty$ for each $i \in V$.

In a network with dynamically changing topology and multiple time-varying delay, we consider the following input (or protocol)