Applied Mathematics and Computation 244 (2014) 479–492

Contents lists available at ScienceDirect

Applied Mathematics and Computation

journal homepage: www.elsevier.com/locate/amc

A game theoretic approach to modeling undesirable outputs and efficiency decomposition in data envelopment analysis

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ARTICLE INFO

Keywords:
Data envelopment analysis
Undesirable outputs
Technical efficiency
Ecological efficiency
Process environmental quality efficiency

ABSTRACT

The changing economic conditions have challenged many organizations to search for more effective performance measurement methods. Data envelopment analysis (DEA) is a widely used mathematical programming approach for comparing the inputs and outputs of a set of homogeneous decision making units (DMUs) by evaluating their relative efficiency. Performance measurement in the conventional DEA is based on the assumptions that inputs should be minimized and outputs should be maximized. However, there are circumstances in real-world problems where some output variables should be minimized. We consider the concepts of *technical efficiency* (the ratio of the desirable outputs to inputs) and *ecological efficiency* (the ratio of the desirable outputs to undesirable outputs) in DEA. We then introduce a new measure called *process environmental quality efficiency* (the ratio of the inputs to the undesirable outputs) and use game theory to integrate these three different efficiency scores into one overall efficiency score. The cooperative and non-cooperative game theory concepts are used to integrate different efficiency ratios into a linear model. We also present a case study to exhibit the efficacy of the procedures and to demonstrate the applicability of the proposed models.

1. Introduction

Data envelopment analysis (DEA) was developed by Charnes et al. [3] to calculate the efficiency score of homogeneous decision-making units (DMUs). The original DEA models measure the efficiency score of the DMUs in terms of their used inputs and desirable (good) outputs. For example, CCR [3], BCC [1], the slack-based measure (SBM) [22] approach, the additive ([4]) and the range adjusted measure (RAM) [9] method are among the mostly used models of this type. The efficiency score measured by these models is known as the technical or operational efficiency index and is an indicator of how

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http://dx.doi.org/10.1016/j.amc.2014.07.035
0096-3003/

efficiently DMUs use their resources to produce desirable outputs and products. Technical efficiency (TE) is the ratio of the desirable outputs to inputs.

However, there might also be some undesirable by-products like CO_2 that are outputs in the production process. Chemical oxygen demand, sulphur dioxide emission, soot, dust and solid waste are some other examples of undesirable outputs. Some authors have studied the modeling of these factors in DEA. Färe et al. [11], Korhonen and Luptacik [14], Hailu and Veeman [12], Seiford and Zhu [19], Färe and Grosskopf [10], Yang and Pollitt [24] and Sueyoshi and Goto [21,20] have addressed different ways to model undesirable outputs. The models discussed by these authors measure overall efficiency (OE) of the DMUs by simultaneous consideration of the inputs, desirable and undesirable outputs. Overall efficiency can be seen as a combination of the technical and ecological efficiency (ECE) scores. The ecological efficiency can be measured as the ratio of the desirable outputs to undesirable outputs. The ecological efficiency, in short, is a measure that is maximized by creating more value with less environmental damage. Among others, Korhonen and Luptacik [14] addressed different ways to measure all three measures represented by the technical, ecological and overall efficiency scores.

The above mentioned researchers have made great strides in modeling undesirable outputs by considering the trade-off between the inputs and the desirable outputs (technical efficiency measurement) and also the trade-off between the desirable outputs and the undesirable outputs (ecological efficiency measurement). These advances have led to the overall efficiency measurement of DMUs which produce undesirable outputs in their production process. The technical efficiency is measured as the ratio of the weighted sum of the desirable outputs divided by the weighted sum of the inputs, while the ecological efficiency is calculated as the weighted sum of the desirable outputs divided by the weighted sum of the undesirable outputs.

However, our paper introduces a new measure of efficiency called "the process environmental quality efficiency (PEQE)" which is considered to be an overall efficiency measurement of the DMUs. The process environmental quality efficiency m is calculated as the ratio of the inputs to undesirable outputs. The process environmental quality efficiency is used to show how a DMU is efficiently using its resources to produce less undesirable outputs. This approach allows the practicing managers and decision makers to discover improvements in the system and decrease the amount of unwanted and undesirable outputs produced by the DMUs.

As an example, consider a performance evaluation problem with two DMUs. DMU 1 with 2 units of input produces 50 units of undesirable output while DMU 2 with 50 units of input produces 2 units of undesirable output. Both DMUs equally produce 1 unit of desirable output. This information suggests that there could be a problem with the quality of the fuel, technology or the process used by DMU 1 which leads to a relatively high emission of the pollution. DMU 1 can be seen as inefficient, since there is another DMU (DMU 2) which is able to produce less undesirable output with more inputs and the same amount of desirable output. This information can be useful to the management of the DMU. The process environmental quality efficiency uses this information to measure the inefficiency of DMU 1. The inefficiency can be due to the low quality of the inputs or the process which has created a high production of pollution.

The goal of our paper is to develop a model to measure the process environmental quality efficiency of the DMUs and also to integrate the technical, ecological and process environmental quality efficiency scores of the DMUs into a single model. For integration, cooperative and non-cooperative game theory concepts are used.

The main contributions of this paper to the performance measurement problems with undesirable outputs are threefold. For the first time: (1) the new concept called the process environmental quality efficiency is introduced and a new model is developed to measure it; (2) the efficiency score for DEA problems with undesirable outputs is decomposed into three different scores: technical, ecological and process environmental quality efficiency scores; and (3) some approaches are proposed to integrate these three efficiency scores into a single model.

The remainder of this paper is organized as follows. In Section 2, we first introduce a new model to measure the quality of the production process from an environmental perspective (PEQE). We then propose some linear and non-linear models to integrate the three different measures of efficiency into a single model. Finally, we show how to implement the proposed models under a common set of weights (CSW). In Section 3, we present a case study to exhibit the efficacy of the procedures and to demonstrate the applicability of the proposed methods in a real-world case. In Section 4, we present our conclusions and future research directions.

2. Proposed model

2.1. Measuring process environmental quality efficiency

Assume we have n DMUs each consuming m inputs to produce s desirable and p undesirable outputs. The outputs corresponding to indices $1,2,\ldots,s$ are desirable and the outputs corresponding to indices $1,2,\ldots,p$ are undesirable. Let $X \in \boldsymbol{R}_+^{m\times n}$, $\boldsymbol{Y}^g \in \boldsymbol{R}_+^{s\times n}$ and $\boldsymbol{Y}^b \in \boldsymbol{R}_+^{p\times n}$ be the matrices, containing the observed input, desirable and undesirable output measures, respectively. x_{ij}, y_{rj}^g, and y_{kj}^b are the ith input, rth desirable and kth undesirable output of DMU_j, respectively. v_i, u_r, and μ_k are the weights of inputs, desirable and undesirable outputs, respectively. Subscript o indicates the DMU under observation. Model (1), known as the CCR model, developed by Charnes et al. [3] is used to measure the technical efficiency of such a DMU. Model (2) is used to measure the ecological efficiency of the DMU under observation. In this model, Korhonen and Luptacik [14] used the ratio of the weighted sum of the desirable outputs divided by the weighted sum of the undesirable outputs as a measure of ecological efficiency.