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Error estimates and superconvergence of mixed finite element methods for fourth order hyperbolic control problems ☆

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ABSTRACT

In this paper, we investigate the error estimates and superconvergence of the semidiscrete mixed finite element methods for quadratic optimal control problems governed by linear fourth order hyperbolic equations. The state and the co-state are discretized by the order k Raviart–Thomas mixed finite element spaces and the control is approximated by piecewise polynomials of order k $(k \geqslant 0)$. We derive error estimates for both the state and the control approximation. Moreover, we present the superconvergence analysis for mixed finite element approximation of the optimal control problems.

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1. Introduction

Optimal control problems [13] have been extensively used in many aspects of the modern life such as social, economic, scientific and engineering numerical simulations. Because of the wide application of these problems, they must be solved successfully through efficient numerical methods. In the recent years, finite element approximation of the optimal control problems has been an important and hot topic in engineering design work, and has been extensively studied in literature [4,10,11,13,16,20]. For the optimal control problems governed by elliptic or parabolic state equations, a priori error estimates of finite element approximations were studied in, for example, [1,9,12,15,17–19]. There also exist lots of works concentrating on the adaptivity of various optimal control problems, see, e.g., [4,18,15,17,16,9].

In many control problems, the accuracy of the gradient is important in numerical discretization of the coupled state equations. Mixed finite element methods are appropriate for the state equations in such cases since both the scalar variable and its flux variable can be approximated to the same accuracy by using such methods, see, for example, [3]. When the objective functional contains the gradient of the state variable, mixed finite element methods should be used for discretization of the state equation with which both the scalar variable and its flux variable can be approximated in the same accuracy. Recently, in [5–7] the authors have done some primary works on a priori, superconvergence and a posteriori error estimates for linear elliptic optimal control problems by mixed finite element methods.

However, as far as we know there is no error-estimates and superconvergence analysis for the fourth order hyperbolic optimal control problems in the literature that can be applied in mixed finite element method. In this article, we shall

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investigate error estimates and superconvergence of the semidiscrete mixed finite element approximation for fourth order hyperbolic optimal control problems.

The optimal control problems that we are interested in is as follows:

$$\min_{u\in K\in U}\left\{\frac{1}{2}\int_0^T\left(\|\Delta y\|^2+\|y-y_d\|^2+\|u\|^2\right)dt\right\}, \tag{1.1}$$

$$y_{tt}(x,t)+\Delta^2 y(x,t)=f(x,t)+u(x,t),\quad x\in\Omega,\ t\in J, \tag{1.2}$$

$$y(x,t)=0,\quad x\in\partial\Omega,\ t\in J, \tag{1.3}$$

$$y(x,0)=y_0(x),\quad x\in\Omega, \tag{1.4}$$

$$y_t(x,0)=y_1(x),\quad x\in\Omega, \tag{1.5}$$

where the bounded open set $\Omega\subset\mathbf{R}^2$ is a rectangle with the boundary $\partial\Omega$, $J=[0,T]$. Let K be a closed convex set in $U=L^2(J;L^2(\Omega)),\ f,\ y_d\in L^2(J;L^2(\Omega)),\ y_0\in H^1(\Omega)$ and $y_1\in H^1(\Omega)$. K is a set defined by

$$K=\left\{u\in U:\ \int_0^T\int_\Omega u\ dxdt\geqslant 0\right\}.$$

Let $\tilde{y}=-\Delta y,\ \tilde{\boldsymbol{p}}=-\nabla y$ and $\boldsymbol{p}=-\nabla\tilde{y}$, then the optimal control problem (1.1)–(1.4) can be equivalently expressed as

$$\min_{u\in K\in U}\left\{\frac{1}{2}\int_0^T\left(\|\tilde{y}\|^2+\|y-y_d\|^2+\|u\|^2\right)dt\right\}, \tag{1.6}$$

$$y_{tt}(x,t)+\operatorname{div}\boldsymbol{p}(x,t)=f(x,t)+u(x,t),\quad x\in\Omega,\ t\in J, \tag{1.7}$$

$$\boldsymbol{p}(x,t)=-\nabla\tilde{y}(x,t),\quad x\in\Omega,\ t\in J, \tag{1.8}$$

$$\operatorname{div}\tilde{\boldsymbol{p}}(x,t)=\tilde{y}(x,t),\quad x\in\Omega,\ t\in J, \tag{1.9}$$

$$\tilde{\boldsymbol{p}}(x,t)=-\nabla y(x,t),\quad x\in\Omega,\ t\in J, \tag{1.10}$$

$$y(x,t)=0,\quad x\in\partial\Omega,\ t\in J, \tag{1.11}$$

$$y(x,0)=y_0(x),\quad x\in\Omega, \tag{1.12}$$

$$y_t(x,0)=y_1(x),\quad x\in\Omega. \tag{1.13}$$

In this paper, we adopt the standard notation $W^{m,p}(\Omega)$ for Sobolev spaces on Ω with a norm $\|\cdot\|_{m,p}$ given by $\|v\|_{m,p}^p=\sum_{|\alpha|\leqslant m}\|D^\alpha v\|_{L^p(\Omega)}^p$, a semi-norm $|\cdot|_{m,p}$ given by $|v|_{m,p}^p=\sum_{|\alpha|=m}\|D^\alpha v\|_{L^p(\Omega)}^p$. We set $W_0^{m,p}(\Omega)=\{v\in W^{m,p}(\Omega):\ v|_{\partial\Omega}=0\}$. For $p=2$, we denote $H^m(\Omega)=W^{m,2}(\Omega),\ H_0^m(\Omega)=W_0^{m,2}(\Omega)$, and $\|\cdot\|_m=\|\cdot\|_{m,2},\ \|\cdot\|=\|\cdot\|_{0,2}$.

We denote by $L^s(0,T;W^{m,p}(\Omega))$ the Banach space of all L^s integrable functions from J into $W^{m,p}(\Omega)$ with norm $\|v\|_{L^s(J;W^{m,p}(\Omega))}=\left(\int_0^T\|v\|_{W^{m,p}(\Omega)}^s dt\right)^{\frac{1}{s}}$ for $s\in[1,\infty)$, and the standard modification for $s=\infty$. For simplicity of presentation, we denote $\|v\|_{L^s(J;W^{m,p}(\Omega))}$ by $\|v\|_{L^s(W^{m,p})}$. Similarly, one can define the spaces $H^1(J;W^{m,p}(\Omega))$ and $C^k(J;W^{m,p}(\Omega))$. The details can be found in [14]. In addition C denotes a general positive constant independent of h.

The plan of this paper is as follows. In Section 2, we shall construct the semidiscrete mixed finite element approximation for the optimal control problems (1.6)–(1.13), then we introduce some projection operators. In Section 3, we derive a priori error estimates of mixed finite element approximation for the control problem. Then the superconvergence analysis is presented in Section 4. Finally, we give a conclusion and some future works.

2. Mixed methods of optimal control problems

In this section, we shall construct the semidiscrete mixed finite element approximation for the optimal control problem (1.6)–(1.13). To fix the idea, we shall take the state spaces $\boldsymbol{L}=L^2(J;\boldsymbol{V})$ and $Q=L^2(J;W)$, where $\boldsymbol{V}$ and W are defined as follows:

$$\boldsymbol{V}=H(\operatorname{div};\Omega)=\left\{\boldsymbol{v}\in(L^2(\Omega))^2,\operatorname{div}\boldsymbol{v}\in L^2(\Omega)\right\},\quad W=L^2(\Omega).$$

The Hilbert space $\boldsymbol{V}$ is equipped with the following norm:

$$\|\boldsymbol{v}\|_{H(\operatorname{div};\Omega)}=\left(\|\boldsymbol{v}\|_{0,\Omega}^2+\|\operatorname{div}\boldsymbol{v}\|_{0,\Omega}^2\right)^{1/2}.$$

Now, we recast (1.6)–(1.13) as the following weak form: find $(\tilde{\boldsymbol{p}},y,\boldsymbol{p},\tilde{y},u)\in(\boldsymbol{L}\times Q)^2\times K$ such that

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