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Special equitorsion almost geodesic mappings of the third type of non-symmetric affine connection spaces

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ABSTRACT

In this paper we investigate a special kind of almost geodesic mapping of the third type of spaces with non-symmetric affine connection. Also we find some relations for curvature tensors of associated affine connection spaces of almost geodesic mappings of the third type. Finally, we investigate equitorsion almost geodesic mapping having the property of reciprocity and find an invariant geometric object of this mapping.

1. Introduction

A lot of research papers and monographs [1–29] are dedicated to the theory of geodesic mappings of Riemannian spaces, affine connected ones and their generalizations. Sinyukov [22] and Mikeš [1,2,12,13,29] gave some other significant contributions to the study of almost geodesic mappings of affine connected spaces.

Let $\mathbb{GA}_{\mathbb{N}}$ be an $\mathbb{N}$-dimensional space with an affine connection L given with the aid of the components L^i_{jk} in each local map V on a differentiable manifold. Generally $L^i_{jk} \neq L^i_{kj}$.

Generalizing the conception of a geodesic mapping for Riemannian and affine connected spaces Sinyukov introduced [22] following notations:

The curve $l : x^h = x^h(t)$ is called an almost geodesic line if its tangential vector $\lambda^h(t) = dx^h/dt \neq 0$ satisfies the equations

$$\bar{\lambda}^h_{(2)} = \bar{a}(t)\lambda^h + \bar{b}(t)\bar{\lambda}^h_{(1)}, \quad \bar{\lambda}^h_{(1)} = \lambda^h_{||\alpha}\lambda^\alpha, \quad \bar{\lambda}^h_{(2)} = \bar{\lambda}^h_{(1)||\alpha}\lambda^\alpha,$$

where $\bar{a}(t)$ and $\bar{b}(t)$ are functions of a parameter t, and $||$ denotes the covariant derivative with respect to the connection on $\bar{\mathbb{A}}_{\mathbb{N}}$.

In the space $\mathbb{GA}_{\mathbb{N}}$, with nonsymmetric affine connection L^i_{jk}, one can define four kinds of a covariant derivative [14,15]. Denote by $|_\theta$ and $||_\theta$ a covariant derivative of the kind $\theta(\theta = 1, 2)$ in $\mathbb{GA}_{\mathbb{N}}$ and $\mathbb{G}\bar{\mathbb{A}}_{\mathbb{N}}$ respectively. For example, for a tensor a^i_j in $\mathbb{GA}_{\mathbb{N}}$ we have

$$a^i_{j\underset{1}{|}m} = a^i_{j,m} + L^i_{\alpha m}a^\alpha_j - L^\alpha_{jm}a^i_\alpha \quad \text{and} \quad a^i_{j\underset{2}{|}m} = a^i_{j,m} + L^i_{m\alpha}a^\alpha_j - L^\alpha_{mj}a^i_\alpha.$$

2. Almost geodesic mappings of non-symmetric affine connected spaces

A mapping f of an affine connected space $\mathbb{A}_{\mathbb{N}}$ onto a space $\bar{\mathbb{A}}_{\mathbb{N}}$ is called an almost geodesic mapping if any geodesic line of the space $\mathbb{A}_{\mathbb{N}}$ turns into an almost geodesic line of the space $\bar{\mathbb{A}}_{\mathbb{N}}$.

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Sinyukov [22] singled out the three types of the almost geodesic mappings, $\pi_1,\ \pi_2,\ \pi_3$ for spaces without torsion. In the present work we investigate the mappings of the type π_3 for spaces with torsion. On a differentiable manifold with nonsymmetric affine connection L^i_{jk}, for a vector exist two kinds of covariant derivative.

Therefore, in the case of the space with nonsymmetric affine connection we can define two kinds of almost geodesic lines and two kinds of almost geodesic mappings.

A curve is called a first kind almost geodesic line if its tangential vector $\lambda^h(t) = dx^h/dt \neq 0$ satisfies the equations

$$\underset{1}{\bar{\lambda}}{}^h_{(2)} = \underset{1}{\bar{a}}(t)\lambda^h + \underset{1}{\bar{b}}(t)\underset{1}{\bar{\lambda}}{}^h_{(1)}, \quad \underset{1}{\bar{\lambda}}{}^h_{(1)} = \lambda^h_{\underset{1}{\|}\alpha}\lambda^\alpha, \quad \underset{1}{\bar{\lambda}}{}^h_{(2)} = \underset{1}{\bar{\lambda}}{}^h_{(1)\underset{1}{\|}\alpha}\lambda^\alpha, \tag{2.1}$$

where $\underset{1}{\bar{a}}(t)$ and $\underset{1}{\bar{b}}(t)$ are functions of a parameter t. A curve is called a second kind almost geodesic line if its tangential vector $\lambda^h(t) = dx^h/dt \neq 0$ satisfies the equations

$$\underset{2}{\bar{\lambda}}{}^h_{(2)} = \underset{2}{\bar{a}}(t)\lambda^h + \underset{2}{\bar{b}}(t)\underset{2}{\bar{\lambda}}{}^h_{(1)}, \quad \underset{2}{\bar{\lambda}}{}^h_{(1)} = \lambda^h_{\underset{2}{\|}\alpha}\lambda^\alpha, \quad \underset{2}{\bar{\lambda}}{}^h_{(2)} = \underset{2}{\bar{\lambda}}{}^h_{(1)\underset{2}{\|}\alpha}\lambda^\alpha, \tag{2.2}$$

where $\underset{2}{\bar{a}}(t)$ and $\underset{2}{\bar{b}}(t)$ are functions of a parameter t.

A mapping f of the space $\mathbb{GA_N}$ onto a space with nonsymmetric affine connection $\mathbb{G\bar{A}_N}$ is called a first kind almost geodesic mapping if any geodesic line of the space $\mathbb{GA_N}$ turns into the first kind almost geodesic line of the space $\mathbb{G\bar{A}_N}$. A mapping f is called a second kind almost geodesic mapping if any geodesic line of the space $\mathbb{GA_N}$ turns into the second kind almost geodesic line of the space $\mathbb{G\bar{A}_N}$.

In this paper, we discuss only the almost geodesic mappings of the first kind.

We can put

$$P^h_{ij} = \bar{L}^h_{ij} - L^h_{ij}, \tag{2.3}$$

where $L^h_{ij},\ \bar{L}^h_{ij}$ are connection coefficients of the space $\mathbb{GA_N}$and $\mathbb{G\bar{A}_N}, (N > 2)$, and P^h_{ij} is a deformation tensor. Then the following theorem is satisfied [24,25,27].

Theorem 2.1. *The mapping f of the space $\mathbb{GA_N}$ onto $\mathbb{G\bar{A}_N}$ is an almost geodesic mapping of the first kind if and only if the deformation tensor $P^h_{ij}(X)$ satisfies identically the conditions*

$$(P^h_{\alpha\beta\underset{1}{|}\gamma} + P^h_{\delta\alpha}P^\delta_{\beta\gamma})\lambda^\alpha\lambda^\beta\lambda^\gamma = \underset{1}{b}\,P^h_{\alpha\beta}\lambda^\alpha\lambda^\beta + \underset{1}{a}\,\lambda^h, \tag{2.4}$$

where $\underset{1}{a}$ and $\underset{1}{b}$ are invariants.

Almost geodesic mappings of the space A_N without torsion are investigated by Sinyukov [22].

The third type almost geodesic mappings of the first kind is determined by condition for the function $b_1(x;\lambda)$ in (2.4):

$$\underset{1}{b} = \frac{\underset{1}{b}_{\alpha\beta\gamma}\lambda^\alpha\lambda^\beta\lambda^\gamma}{\sigma_{\varepsilon\delta}\lambda^\varepsilon\lambda^\delta},$$

where $\sigma_{\varepsilon\delta}\lambda^\varepsilon\lambda^\delta \neq 0$. Then for the deformation tensor we get [25]

$$P^h_{ij}(x) = \psi_i(x)\,\delta^h_j + \psi_j(x)\,\delta^h_i + \sigma_{ij}(x)\varphi^h(x) + \xi^h_{ij}(x). \tag{2.5}$$

Here ξ^h_{ij} is an anti-symmetric tensor of the type $\begin{pmatrix}1\\2\end{pmatrix}$. From (2.5) and (2.4) we have

$$\varphi^h_{\underset{1}{|}m} + \xi^h_{\varepsilon m}\varphi^\varepsilon = \nu_m\varphi^h + \mu\delta^h_m. \tag{2.6}$$

Here ν_m is a covariant vector, μ is an invariant and ξ^h_{ij} is an anti-symmetric tensor.

The Eqs. (2.5) and (2.6) characterize the third type almost geodesic mapping of the first kind. That mapping we denote by $\underset{1}{\pi_3}$.

Let the mapping $f : \mathbb{GA_N} \to \mathbb{G\bar{A}_N}$ have *the property of reciprocity*, i.e. f and its inverse are of type $\underset{1}{\pi_3}$. Then the following condition is satisfied

$$\varphi^h_{\underset{1}{\|}m} - \xi^h_{\alpha m}\varphi^\alpha = \tilde{\nu}_m\varphi^h + \tilde{\mu}\delta^h_m,$$

where $\tilde{\nu}_m$ is a vector and $\tilde{\mu}$ is an invariant.

From here follows (see [25])

$$\xi^h_{\alpha m}\varphi^\alpha = \theta_m\varphi^h + \bar{\rho}\delta^h_m, \tag{2.7}$$

where

$$\theta_m = \nu_m - \tilde{\nu}_m + \sigma_{\alpha m}\varphi^\alpha + \psi_m, \quad \bar{\rho} = \mu - \tilde{\mu} + \psi_\alpha\varphi^\alpha.$$

Here $\bar{\rho}$ is an invariant, and θ_m is a vector.

Suppose that the conditions (2.7) are satisfied identically with respect to φ^h. Then we have a special class of almost geodesic mappings $\underset{1}{\tilde{\pi}_3}$. The basic equations which give the characterization of this mapping [25] has the form