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Effect of coupled radial and axial variability of viscosity on the peristaltic transport of Newtonian fluid

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ABSTRACT

Many authors assume viscosity to be constant or a radius exponential function in Stokes' equations in order to study the peristaltic motion of a Newtonian fluid through an axisymmetric conduit. In this paper, viscosity is assumed to be a function of both the radius and the axial coordinate. More precisely, it is dependent on the distance from the deformed membrane given the fact that the change in viscosity is caused by the secretions released from the membrane. The effect of this hypothesis on the peristaltic flow of a Newtonian fluid in axisymmetric conduit is investigated under the assumptions of long wavelength and low Reynolds number. The expressions for the pressure gradient and pressure rise per wavelength are obtained and the pumping characteristics and the phenomena of reflux and trapping are discussed. We present a detailed analysis of the effects of the variation of viscosity on the fluid motion, trapping and reflux limits. The study also shows that, in addition to the mean flow parameter and the wave amplitude, the viscosity parameter also affects the peristaltic flow. It has been noticed that the pressure rise, the friction force, the pumping region and the trapping limit are affected by the variation of the viscosity parameter but the reflux limit and free pumping are independent of it.

1. Introduction

Several works have analyzed the mechanics of peristaltic pumping of a Newtonian fluid through an axisymmetric conduit. One of the remaining works is the treatment of the Stokes equation under an infinitely long wavelength approximation and a negligible Reynolds number in order to evaluate the velocities and pressures at each point of the geometrical model for a viscous fluid. Most works used the sine function for the geometry of the intestinal wall surface and a constant or an exponential function or its linear approximation for viscosity. For example, Latham [1] was the first to investigate the mechanism of peristalsis in relation to mechanical pumping. Shapiro et al. [2] studied the peristaltic motion of Newtonian fluid with constant viscosity, under long-wavelength and low-Reynolds number assumptions, for plane and axisymmetric flow. They discussed the pumping characteristics and the phenomena of reflux and trapping. Shukla et al. [3] investigated the effects of peripheral-layer viscosity on peristaltic transport of a bio-fluid in uniform tube, the shape of interface is specified by them independently of the fluid viscosities. The invalidity of their analysis was proved by Brasseur et al. [4] with the limit of infinite peripheral layer viscosity, since the conservation of mass is not satisfied. They presented the effect of the peripheral layer on the fluid motion and discussed the pumping characteristics and the phenomena of reflux and trapping. They concluded that, for a peristaltic motion, a peripheral layer more viscous than the core fluid improves the pumping performance,

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while a less-viscous peripheral layer degrades performance. Elshehawey et al. [5] studied the peristaltic motion of an incompressible Newtonian fluid through a channel in the presence of three layers flow with different viscosities. Srivastava et al. [6,7], discuss the pressure rise, peristaltic pumping, augmented pumping and friction force for Newtonian fluid on peristaltic motion, with variable viscosity in uniform and non-uniform tubes and channel under zero Reynolds number and long wavelength approximations and comparison with other theories are given. Their computational results indicate that the pressure rise decreases as the fluid viscosity decreases. The difference between two corresponding values (for constant and variable viscosity) of the pumping region increases with increasing amplitude ratio. Further, the free pumping increases as viscosity of fluid decreases. The pressure rise, in the case of non-uniform geometry is found to be much smaller than the corresponding value in the case of uniform geometry. Abd El Hakim et al. [8,9] have investigated the effect of endoscope and fluid with variable viscosity on peristaltic motion. Abd El Hakim et al. [10] choose the viscosity parameter in the exponential function as a perturbation parameter in order to study the hydromagnetics flow of fluid with variable viscosity in a uniform tube with peristalsis under a negligible Reynolds number, a small magnetic Reynolds number, sine harmonic traveling wave and infinitely long wavelength approximation. The effect of variable viscosity on the peristaltic transport of a Newtonian fluid in an asymmetric channel has been studied by Hayat and Ali [11]. They showed that, in addition to the effect of mean flow parameter, the wave amplitude also affects the peristaltic flow. Recently a number of analytical, numerical and experimental studies of peristaltic flows of different fluids have been reported. Rathod and Asha [12] studied the effects of magnetic field and an endoscope on peristaltic motion and Rathod and Devindrappa [13] studied the slip effect on peristaltic transport of a porous medium in an asymmetric vertical channel by Adomian decomposition method. Hina et al. [14] studied the Slip Effects on magnetohydrodynamic Peristaltic Motion with Heat and Mass Transfer. A mathematical model has been developed by Misra and Maiti [15] with an aim to study the peristaltic transport of a rheological fluid for arbitrary wave shapes and tube lengths.

The previously mentioned works are based on Newtonian fluids with a viscosity that is either constant or function of the radius only. This is inconsistent with the situation in natural peristaltic motions. For instance, chyme motion within the small intestine is subjected to water intake at a first stage, then to water absorption accompanied by some acids released from the intestinal membrane to facilitate digestion and enable the absorption of necessary nutrients such as proteins, fat and vitamins [16–18]. The movement of blood in the blood vessels is characterized by a concentration of red blood cells near the axis and of white blood cells and plasma near the membrane, which makes the viscosity decrease closer to the wall [19]. This shows that viscosity depends on the distance from the membrane in its deformed state, which again contradicts the assumptions of constant or radially varying viscosity since these imply a constant viscosity along lines parallel to the axis. This led us in this work to consider the viscosity to be a function of two variables. We assume that the viscosity remains constant on any surface such that the radius is proportional to that of the membrane with a factor less than unity. This choice is consistent with the observation mentioned above concerning the distribution of viscosity in the blood vessels and the small intestine. It is expected to provide more accurate and realistic explanations of the mechanisms of peristaltic movement and blood circulation and the associated reflux and trapping phenomena. Moreover, in order to make this work even more general an additional parameter has been introduced into the viscosity function. This parameter indicates the gradient of the viscosity. We further studied the effect of this parameter on pumping, reflux and trapping and compared it with published results based on the assumptions of viscosity being either constant or function of a single variable.

2. Formulation and analysis

We consider the creeping flow of an incompressible Newtonian fluid with variable viscosity through an axisymmetric form in a uniform tube wall thickness with a wave traveling down its wall. We use the cylindrical coordinate system

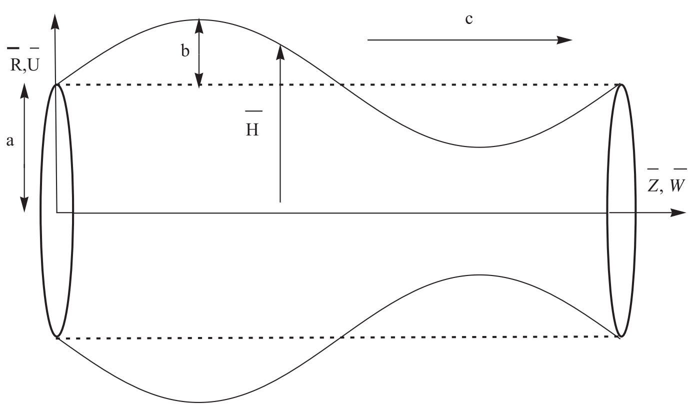

Fig. 1. Peristaltic transport through a tube.