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An evidential game theory framework in multi-criteria decision making process

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ABSTRACT

Multi-criteria decision making process is a widely concerned issue. In traditional studies, it is assumed that a decision maker's behaviors are not affected by others. However, competitive environment is more general in real world where everybody maximizes his benefits and each people is inevitably influenced by others' behaviors. In this paper, an evidential game theory framework is proposed in order to address the multi-criteria decision making process in the competitive environment. The proposed framework includes five stages, namely game analysis, decision-making analysis, strategies evaluations, fusion of evaluations, decision based on equilibrium, respectively. Within the proposed framework, a representation form of uncertain information called belief structure derived from Dempster–Shafer theory is employed to model the uncertainty involving experts' subjective evaluations. The game theory is used to find the optimal combination of strategies in the interactive decision situations. The application of the proposed evidential game theory framework is given by an illustrative example.

1. Introduction

The decision making is a complicated process to select a proper alternative among various options based on their performance of evaluation on various criteria including benefit criteria and cost criteria. In terms of the objectives and environment, the decision making includes many different types, such as group decision making [1], multi-objective decision making [2,3], and so on [4–7]. A lot of methods are developed to address the decision making process and obtain the most proper alternative [8–12]. Due to the complexity and uncertainty of environment, the decision making process has been greatly concerned by many researchers.

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In the decision making process, the representation of evaluation is always concerned by researchers [13]. During the evaluation process, the assessment results are often given by domain experts. However, due to the subjective judgments caused by the lack of information or the expert's inability to provide accurate judgements, it is inevitable to involve the subjective judgements of human beings. The primary challenge is how to model the uncertainty involving expert's assessment. In many studies, fuzzy set theory [14–16] is often regarded as an effective means to represent the uncertainty of human begins' judgement. The linguistic variables represented by fuzzy numbers can highly models human beings' mind, such as "the quality is good", "the safety is high", etc. The fuzzy set theory provides a mathematical framework to systematically process linguistic information. However, it is lack of directly expressing the uncertainty involving fuzzy and probabilistic simultaneously. For example, "the car is good with a certainty of 0.8", "it is 0.7 sure that the weather is sunny and 0.3 sure that the weather is cloudy", etc. Facing that situation, belief structure is extensively used to better represent the uncertainty involving human beings' judgement in many fields [17–21].

Belief structure, also called basic probability assignment or mass function, is a basic representation form of information in Dempster–Shafer theory [22,23]. Because of the ability to express uncertain information and synthesize all the knowledge of the belief structures, as a knowledge reasoning tool, Dempster–Shafer theory, or evidence theory, has been widely applied to many fields such as pattern recognition [24,25], identification of influential nodes [26,27], risk assessment [28,29], etc [30–38]. With the use of belief structures, a more effective framework can be built in the decision making process involving various types of uncertainty.

Regardless of the types of decision, in the traditional multi-criteria decision making process, each decision maker's behavior is not affected by others. In that situation, much attention is mostly paid on the evaluation of alternatives with respect of various criteria. However, competitive environment is more general in real world that somebody maximizes his benefits and interacts with others. All of these activities involving more than one person could be classified in same aspect, which are called games [39]. Game theory provides a mathematical tool for this study of games, ranging from behavioral economics [40,41] to evolutionary game [42–46], and so on [47–49]. In a sense, the classical decision making process can be seen as a special game where the decision makers play games with nature to maximize their benefits. The study of multi-criteria decision making process from the perspective of game theory has attracted much attention [50–55].

Depending on the advantage of belief structure to represent uncertain information, the aim of this paper is to establish an evidential game theory framework to study the multi-criteria decision making process in the competitive environment where a decision maker must consider other's behavior before making decisions. Within the proposed evidential game theory framework, the belief structure is used to represent the qualitative and quantitative judgements given by experts. And these judgements from multi-criteria and multi-experts are fused into a comprehensive evaluation. Finally, the equilibrium of decision makers is found based on game theory.

The rest of this paper is organized as follows. Section 2 gives a brief introduction about the two-person non-constant sum game and Dempster–Shafer theory. In Section 3, the proposed evidential game theory framework is presented. In Section 4, the application of the proposed evidential game theory framework is given by using an illustrative example. Section 5 concludes this paper.

2. Preliminaries

Some background knowledge about two-person non-constant sum game and Dempster–Shafer theory are introduced in this section.

2.1. Two-person non-constant sum game

Game theory provides a mathematical framework to explain and address the interactive decision situations where the aims, goals and preferences of the participating agents are potentially in conflict [56–61]. Due to its advantages to describe interactions between agents, game theory is widely applied from economics [62,63], biology [64–66], to evolution of cooperation [67–69], as well as other disciplines [70–72]. A strategic game contains three parts: set of players, set of strategies for each player and payoff for each strategy combination, respectively. A solution to a game is a certain combination of strategies. This solution which is self enforcing and no player can gain by unilaterally deviating from it, is said to be a Nash equilibrium.

Two-person non-constant sum game is a kind of widely addressed game, such as iterated prisoner's dilemma game. In this game, player 1 has a finite strategy set S_1 including p strategies. Player 2 has a finite strategy set S_2 including q strategies. The payoffs of player 1 and player 2 are determined by functions $u_1(s_1, s_2)$ and $u_2(s_1, s_2)$, respectively, where $s_1 \in S_1$ and $s_2 \in S_2$. A combination of each players' strategy (s_1^*, s_2^*) is a Nash equilibrium of this two-person non-constant sum game if

$$u_1(s_1^*, s_2^*) \geqslant u_1(s_1, s_2^*) \quad \forall s_1 \in S_1, \tag{1}$$

$$u_2(s_1^*, s_2^*) \geqslant u_2(s_1^*, s_2) \quad \forall s_2 \in S_2. \tag{2}$$