Applied Mathematics and Computation 244 (2014) 903–911

Contents lists available at ScienceDirect

Applied Mathematics and Computation

journal homepage: www.elsevier.com/locate/amc

Iterative solutions for a coupled system of fractional differential–integral equations with two-point boundary conditions ☆

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A R T I C L E I N F O

Keywords:
Fractional differential systems
Boundary value problems
Monotone iterative technique
Upper and lower solutions

A B S T R A C T

In this paper, we deal with a coupled system of fractional differential–integral equations with two-point boundary conditions. The maximal and minimal solutions are obtained by using the monotone iterative technique combined with the method of upper and lower solutions, meantime the error estimate of the solutions are given. In the end, some examples are given to illustrate the results.

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1. Introduction

Fractional calculus has gained considerable popularity due to its frequent appearance in various fields such as physics, aerodynamics, electrodynamics of complex medium, polymer rheology, etc. For details, see [1–3] and the references therein. And there has been a significant development in fractional differential equations in recent years.

On the other hand, the study of coupled systems involving fractional differential equations is also important as such systems occur in various problems of applied nature (see [4–7]). So considerable work has been done to study the existence result for them nowadays (see [8–12]). And the authors got the existence solutions by the method of fixed-point theorem or coincidence degree theorem.

The monotone iterative technique, combined with the method of upper and lower solutions, is a powerful tool for proving the existence of solutions of nonlinear differential equations, for instance, see [13–15] and the references therein. In [16–18], the authors used the method of upper and lower solutions investigated the existence of solutions for initial value problems with fractional differential equations. By the same method some people got the solutions of boundary value problems for fractional differential equations, such as [19–21]. To the best of our knowledge, only few papers (such as [22,23]) considered the method of upper and lower solutions for boundary value problems with fractional coupled systems.

Motivated by [22,23], in this paper, we use the monotone iterative technique, combined with the method of upper and lower solutions to study the coupled system of fractional differential–integral equations with two-point boundary conditions, which is given by

☆ This work is sponsored by NNSF of China (11271364).

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http://dx.doi.org/10.1016/j.amc.2014.07.043
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$$\begin{cases} D^{\alpha}u(t) + f(t, v(t), I^{\beta}v(t)) = 0, & t \in [0,1], \\ D^{\beta}v(t) + g(t, u(t), I^{\alpha}u(t)) = 0, & t \in [0,1], \\ I^{3-\alpha}u(t)|_{t=0} = D^{\alpha-2}u(t)|_{t=0} = u(1) = 0, \\ I^{3-\alpha}v(t)|_{t=0} = D^{\alpha-2}v(t)|_{t=0} = v(1) = 0, \end{cases} \tag{1.1}$$

where $2 < \alpha, \beta \leq 3, u(t), v(t) \in C[0,1], f, g : I \times \mathbf{R} \times \mathbf{R} \mapsto \mathbf{R}$ satisfy Carathéodory conditions. D^{α}, D^{β} and I^{α}, I^{β} are the standard Riemam–Liouville fractional derivative and fractional integral respectively.

As applying, we get the existence of solutions for a coupled system of fractional differential equations with two-point boundary problems

$$\begin{cases} D^{\alpha}u(t) + f(t, v(t)) = 0, & t \in [0,1], \\ D^{\beta}v(t) + g(t, u(t)) = 0, & t \in [0,1], \\ I^{3-\alpha}u(t)|_{t=0} = D^{\alpha-2}u(t)|_{t=0} = u(1) = 0, \\ I^{3-\alpha}v(t)|_{t=0} = D^{\alpha-2}v(t)|_{t=0} = v(1) = 0, \end{cases} \tag{1.2}$$

where $2 < \alpha, \beta \leq 3, u(t), v(t) \in C[0,1], f, g : I \times \mathbf{R} \mapsto \mathbf{R}$ satisfy Carathéodory conditions.

The rest of this paper is organized as follows. In Section 2, we give some necessary notations, definitions and lammas. In Section 3, we establish theorems of solutions for the problems (1.1) and (1.2). In Section 4, we give two examples to demonstrate our result.

2. Preliminaries

In this section, we present some definitions, lemmas and assumptions that will be used in the whole paper, and the definitions about fractional calculus theory can be found in [1–3].

Definition 2.1. Let $\alpha > 0$, the operator I^{α}, defined on $L[a,b]$ by

$$_aI_t^{\alpha}u(t) = \frac{1}{\Gamma(\alpha)} \int_a^t (t-s)^{\alpha-1} u(s)\,ds$$

for $a \leqslant t \leqslant b$, is called the Riemann–Liouville fractional integral operator of order α, where $a \in R$ and Γ is the Gamma function.

Definition 2.2. Let $\alpha > 0$ and $n = [\alpha] + 1$, the operator D^{α}, defined by

$$_aD_t^{\alpha}u(t) = \frac{1}{\Gamma(n-\alpha)} \left(\frac{d}{dt}\right)^n \int_a^t (t-s)^{n-\alpha-1} u(s)\,ds$$

is called the Riemann–Liouville fractional differential operator of order α.

Definition 2.3 [24]. Let E and F be partially ordered Banach spaces, $D \subset E, A : D \mapsto F$ be an operator. For all $x, y \in D$ with $x \leqslant y$, if we have $Ax \leqslant Ay$, then A is called an increasing operator.

Definition 2.4 [24]. Let E be a Banach space, $D \subset E, A : D \mapsto F$ be an operator. If $x_0 \leqslant Ax_0$ (resp., $x_0 \geqslant Ax_0$) holds for all $x_0 \in D$, then x_0 is called the lower (resp., upper) solution of operator equation $x = Ax$.

Definition 2.5 [24]. We say the function $f(t, u, v) : [0,1] \times \mathbf{R} \times \mathbf{R} \mapsto \mathbf{R}$ satisfies Carathéodory conditions, if the following conditions are satisfied:

(1) for each $u, v \in R, f(t, u, v)$ is Lebesgue measurable for t;
(2) for almost every $t \in [0,1], f(t, u, v)$ is continuous for u, v.

Definition 2.6 [25]. Functions $(u_0(t), v_0(t)) \in C[0,1] \times C[0,1]$ are called a lower solution of (1.1) if they satisfy

$$\begin{cases} D^{\alpha}u_0(t) + f(t, v_0(t), I^{\beta}v_0(t)) \leqslant 0, & t \in (0,1), \\ D^{\beta}v_0(t) + g(t, u_0(t), I^{\alpha}u_0(t)) \leqslant 0, & t \in (0,1), \\ I^{3-\alpha}u_0(t)|_{t=0} \leqslant 0, D^{\alpha-2}u_0(t)|_{t=0} \leqslant 0, & u_0(1) \leqslant 0, \\ I^{3-\alpha}v_0(t)|_{t=0} \leqslant 0, D^{\alpha-2}v_0(t)|_{t=0} \leqslant 0, & v_0(1) \leqslant 0. \end{cases}$$

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